

Searches for skyrmions in the limit of zero g -factor

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Abstract. Energy gaps have been measured for the ferromagnetic quantum Hall effect states at $\nu = 1$ and 3 in GaAs/Ga_{0.7}Al_{0.3}As heterojunctions as a function of Zeeman energy, which is reduced to zero by applying hydrostatic pressures of up to 20 kbar. At large Zeeman energy the gaps are consistent with spin-wave excitations. For a low-density sample the gap at $\nu = 1$ decreases with increasing pressure and reaches a minimum when the g -factor vanishes. At small Zeeman energy the excitation appears to consist of a large number of reversed spins and may be interpreted as a skyrmion. The data also suggest that skyrmionic excitations take place at $\nu = 3$. The width of the minimum at $\nu = 1$ is found to decrease as the g -factor is reduced in a similar way for all samples.

1. Introduction

The magnetic fields at which the integer quantum Hall effect (IQHE) [1] occurs correspond to the formation of a strongly interacting electron gas which can have a number of novel and interesting excitations. This is particularly true when an odd number of levels are occupied, which makes the system spin polarized. At $\nu = 1$ (where the filling factor $\nu = n_e \hbar / eB$ measures how many Landau levels (LLs) are filled) the ground state should be regarded as a ferromagnet since all the spin-down states in the lowest LL are occupied while all the spin-up states are empty. In GaAs the single-particle (SP) Zeeman energy (ZE) $g\mu_B B$ is very small $\sim 0.3 \text{ K T}^{-1}$ and Coulomb interactions are very significant. This has led several authors to suggest that novel charged excitations with non-trivial spin order, known as charged spin-texture excitations or skyrmions, may occur [2, 3].

In this paper we report experiments that investigate the nature of the collective excitations in the region of vanishing Zeeman energy, where the Zeeman energy is controlled by the use of hydrostatic pressure. Initially we will explain what is meant by and examine the difference between spin waves and skyrmions. The high-pressure experiments will then be described and results presented for $\nu = 1$ that under different conditions show excitation of spin waves and large skyrmions. Attention will then turn

to $\nu = 3$ where the data also suggest skyrmionic excitations at vanishing ZE.

The IQHE occurs when the Fermi energy is in a mobility gap of the electronic density of states and at low temperature this leads to quantized plateaux in the off-diagonal component of magnetoresistivity ρ_{xy} and zeros in ρ_{xx} at integer filling factors ν . The temperature dependence of these resistivity components can be used to measure the size of the energy gaps E_g . For even-integer ν the gaps correspond to the cyclotron energy $\hbar\omega_c$ arising from the orbital motion of electrons, which experiments correctly measured as 20 K T^{-1} in GaAs. At odd integers the SP ZE is tiny compared to $\hbar\omega_c$, yet experimentally the odd and even IQHEs appear in a similar temperature regime. At finite temperature the depth of an IQHE minimum in ρ_{xx} is determined by smearing of the Fermi function and it will be approximately 50% developed when $kT \sim E_g/6$. Thus if the energy gap for odd IQHE were determined by a SP ZE it would only be observable for $B/T > 20$, requiring $T < 50 \text{ mK}$ to observe the spin splitting below 1 T and 80 T to see $\nu = 1$ at 4.2 K. This is clearly at variance with transport experiments, which always measure a much larger gap [4], and suggests that the excitations at odd-integer ν are instead due to a collective motion within the two-dimensional electron gas (2DEG). The energy scale of these excitations can be expected to scale with the

Coulomb energy [5], and the resulting increased splitting is sometimes referred to as an exchange enhanced g -factor.

The $\nu = 1$ ferromagnetic ground state is significantly different from the more familiar Heisenberg ferromagnet since the spontaneous magnetization occurs in the presence of a quantizing magnetic field, not at zero field, and the spins associated with the charge carriers are free to move, hence it is termed an ‘itinerant’ ferromagnet [2, 3]. Two types of charged excitations from the ferromagnetic ground state that produce a well separated spin-up electron and spin-down hole have been identified. The first is a spin wave (really a spin exciton) [5] whose energy depends on the wavevector, usually given by the dimensionless quantity kl_B where $l_B = \sqrt{\hbar/eB}$ is the magnetic length. At long wavelength, i.e. $kl_B = 0$, corresponding to no spatial separation between the electron and the hole, the spin-wave energy is equal to $g\mu_B B$ as measured in spin resonance experiments [6]. Transport measurements are sensitive to the opposite limit where the electron and hole are well separated, i.e. large kl_B , and see the whole Coulomb exchange energy $E_c = e^2/4\pi\epsilon l_B$ which in GaAs is $50.55\sqrt{B}$ K, much larger than the SP ZE and closer to $\hbar\omega_c$. The energy to create a spin wave is $E_{sw} = g\mu_B B + \kappa E_c$, where κ is the spin stiffness calculated to be $\sqrt{\pi/2}$ in the ideal case.

The second type of excitation is based on a spin texture that consists of a central reversed spin surrounded by rings of spin that gradually cant over until at the edge they are aligned with the external magnetic field. We will refer to such spin textures as skyrmions, although strictly this term is reserved for objects of infinite extent at zero ZE. The essential differences between this two-dimensional spin texture and a spin wave are that the net spin may be greater than one and on a path taken around the central spin there will be a change of spin orientation equivalent to a winding number of unity. In a system with zero ZE the skyrmions should have infinite extent but for finite ZE they have a finite size that can be characterized by the number of reversed spins R contained in the skyrmion. When the filling factor moves away from $\nu = 1$ the ground state will contain a number of skyrmions (quasi-holes) or anti-skyrmions (quasi-electrons) and this has been detected from the degree of spin polarization in nuclear magnetic resonance [7] and photoluminescence [8] experiments. Both of these measurements suggest that $R \sim 7$. In transport measurements at exactly $\nu = 1$ the excitations consist of well separated skyrmion–anti-skyrmion pairs which, for infinite skyrmions, only cost half the exchange energy required for a large spin exciton, but R times the ZE. One way to think of this is that the spin texture of the skyrmions dresses the spin exciton. Eventually, at large enough ZE, $R = 1$ and skyrmions are indistinguishable from the undressed spin excitons.

The balance between the SP ZE and the Coulomb energy is determined by the parameter $\eta = g\mu_B B/E_c$ which determines whether skyrmions with $R > 1$ (small $|\eta|$) or spin waves (large $|\eta|$) will be the lowest-lying excitations [9]. The crossover is calculated to be at $|\eta| = 0.054$ [2]. It should be noted that $\eta \propto \sqrt{B}$ so skyrmionic excitations are expected to be favoured at low

magnetic fields and small g -factors. To date, two transport measurements have inferred the existence of skyrmions. Increasing $|\eta|$ by tilting the magnetic field suggested a seven-spin excitation for $|\eta| \sim 0.01$ [10]. In a narrow quantum well where g is already reduced by penetration of the wavefunction into the AlGaAs barrier, it was decreased further by hydrostatic pressure, becoming zero at 4.8 kbar where the energy gap at $\nu = 1$ showed a minimum [11]. This indicated a much larger skyrmionic excitation consistent with $R = 33$ when $|\eta| < 0.002$.

The ground states at higher odd filling factors will also be ferromagnetic but the effect on transport measurements is expected to be less pronounced as only a fraction $1/\nu$ of the electrons is involved in the collective motion. The remainder in full LLs do not contribute to the transport current, but may act to screen any Coulomb interactions. Skyrmions also appear in the excitation spectrum at higher odd filling factors but for an ideal 2DEG they are calculated to have higher energies than the single spin exciton at vanishing ZE [12]. However, when the finite thickness z of a real 2DEG is taken into account, skyrmions may become the lowest-energy excitation at $\nu = 3$ [13] and also at all other odd filling factors for sufficiently extended wavefunctions [14]. The stability and size of the skyrmions is predicted to increase with z but to be reduced by finite ZE and LL mixing. For example, at $\eta = 0$ skyrmions become the lowest excitation at $\nu = 3$ once $z > 0.1l_B$, while for $z = l_B$ the transition occurs at $|\eta| = 0.0037$. So, although skyrmions were not observed in [10] with $|\eta| > 0.007$ or in [15], where the sample mobilities were relatively low and measurements were done at high fields, this does not preclude their existence under more favourable conditions.

2. High-pressure experiments

This paper reports experiments in which skyrmion formation is favoured by reducing the g -factor. This is achieved by applying hydrostatic pressures of up to 22 kbar [16]. In GaAs at ambient pressure $g = -0.44$, as a result of subtracting band-structure effects driven by the spin–orbit interaction from the free electron value of 2. At higher pressure the band-structure contribution reduces, and so does the magnitude of g which passes through zero at ~ 18 kbar. The pressure for this zero crossing decreases slightly at higher magnetic field, as the cyclotron energy increases the energy separation between the electron and hole bands. The g -factor has been calculated using $\mathbf{k} \cdot \mathbf{p}$ theory [17, 18] and may be approximated by the following expression

$$g = 2 - 19300 \left(\frac{1}{1519 + \hbar\omega_c + 10.7P} - \frac{1}{1860 + \hbar\omega_c + 10.7P} \right) - 0.12 \quad (1)$$

where P is the pressure in kbar and all far-band terms have been assumed to stay constant [18].

The samples used in this study were high-quality GaAs/Ga_{0.7}Al_{0.3}As heterojunctions grown by molecular beam epitaxy at Philips Research Laboratories, Redhill. Samples G586, G627 and G902 have undoped spacer layers

of 40, 40 and 20 nm. At ambient pressure and 4 K their respective electron densities n_e after photoexcitation are 3.3 , 3.5 and $5.7 \times 10^{15} \text{ m}^{-2}$ with corresponding mobilities of 300 , 370 and $200 \text{ m}^2 \text{ V}^{-1} \text{ s}^{-1}$. The samples were mounted inside a non-magnetic beryllium copper clamp cell [19] and the pressure was measured from the resistance change of manganin wire. The absolute values quoted at low temperature are accurate to ± 1 kbar, but between data points the variation is less than ± 0.2 kbar. The pressure cell was attached to a top-loading dilution refrigerator probe, allowing temperatures as low as 30 mK to be obtained and measured with a ruthenium oxide resistor attached outside the pressure cell, which followed the sample temperature with a negligible time lag. Experiments were also performed at temperatures up to 15 K using a separate variable-temperature cryostat.

Increasing the pressure causes the GaAlAs conduction band to move relative to the GaAs conduction band in the well, reducing the number of electrons. Above ~ 13 kbar no electrons were present in the dark at low temperature, but a certain number could be recovered after illumination from a red light-emitting diode (LED). The illumination time required to obtain a constant number of electrons roughly doubled for every 2 kbar increase in pressure, reaching several hours at 20 kbar. The highest pressure studied was 22 kbar, but no conductivity could be measured despite prolonged illumination. The sample required several hours for the density to stabilize before quantitative measurements could be made during which it varied by less than 1% over the full temperature range. The data from G586 were recorded with a density of $0.44 \pm 0.06 \times 10^{15} \text{ m}^{-2}$ above 13 kbar and slightly higher at lower pressures. This puts $\nu = 1$ at 1.8 T. For G627 and G902 the data were recorded over the wider density range $0.8\text{--}3.3 \times 10^{15} \text{ m}^{-2}$, i.e. $\nu = 1$ between 3 T and 14 T, but only for pressures up to 14 kbar using a Polish pressure cell and an InSb pressure gauge.

Skyrmion formation is favoured by both lower magnetic fields and higher pressures, which reduce $|\eta|$. In order to study this regime, data from sample G586 were taken for a constant $n_e \sim 0.44 \pm 0.06 \times 10^{15} \text{ m}^{-2}$, and up to higher pressures than the other two samples where $\nu = 1$ is always at larger fields making the reduction of $|\eta|$ harder. Figure 1 shows ρ_{xx} at pressures of 13, 17 and 20 kbar for the temperature range 40–1300 mK.

Values of the energy gaps have been extracted by fitting the temperature dependence of the resistivity minima to the Lifshitz–Kosevitch (LK) formula, which accounts for thermal smearing of the Fermi function. In this formula $\Delta\rho_{xx} \propto X/\sinh X$, where $X = 2\pi^2 kT/E_g$ and $\Delta\rho_{xx}$ is defined as $(\rho_{xx}(\infty) - \rho_{xx}(T))/\rho_{xx}(\infty)$, with $\rho_{xx}(\infty)$ the resistivity that would be observed in the absence of the IQHE. This procedure, described in more detail in [20], has the advantages over finding activation energies from an Arrhenius plot that, firstly, it measures the gap between LL centres not the mobility gap, and so is less sensitive to changes in disorder, and secondly, an accurate zero of resistance is not required, which avoids any problems of parallel conduction and means especially low temperatures are not required. Examples of the fitted data are shown in figure 2. A possible disadvantage of the LK method is that

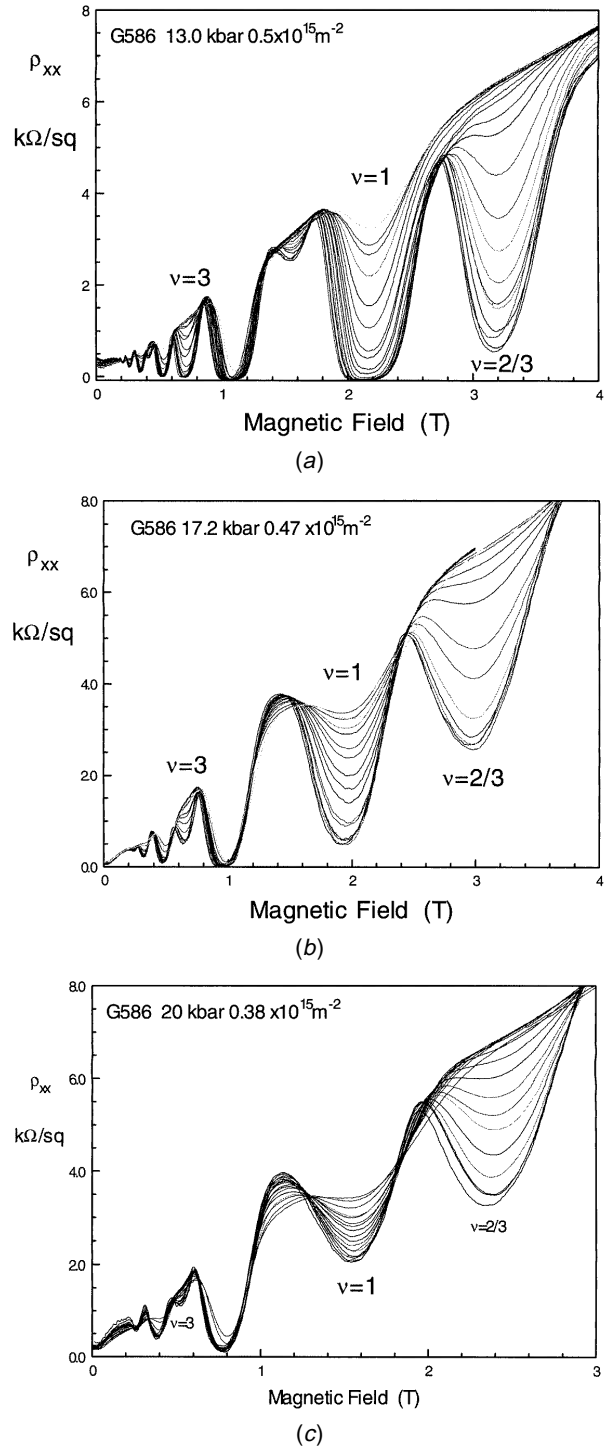


Figure 1. Magnetoresistance for sample G586 between 50 mK and 1.3 K at (a) 13 kbar, (b) 17 kbar and (c) the highest pressure obtained of 20 kbar.

the majority of measurements are made in a temperature range in which the system may not remain totally spin polarized. The accuracy of the LK fitting procedure has been tested by considering the energy gaps at even integers which were found to be within 1% of the expected $\hbar\omega_c$, e.g. in sample G586 at 10 kbar, with $\nu = 1$ at 3.6 T, the gaps at $\nu = 4, 6, 8$ and 10 were 15.7, 10.7, 7.9 and 6.4 K

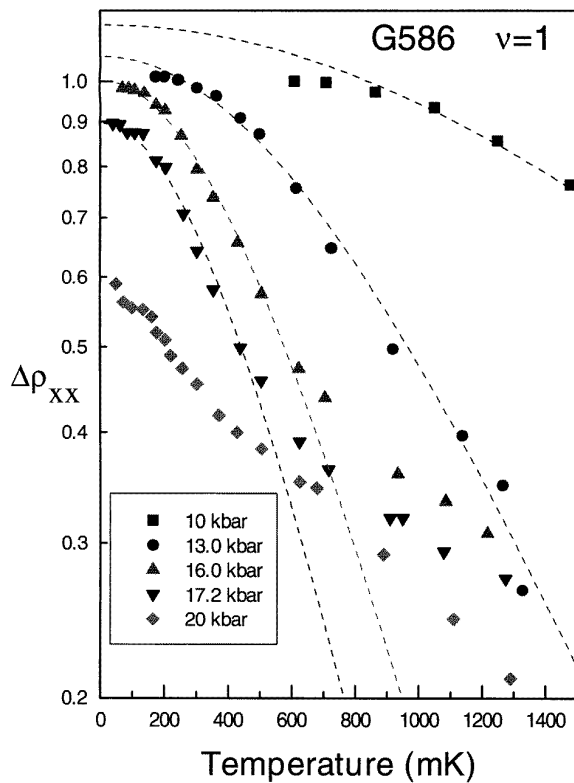


Figure 2. Temperature dependence of the $\nu = 1$ minimum $\Delta\rho$ (as defined in the text) for sample G586 at pressures between 10 and 20 kbar. The dashed curves show fits to the LK formula. (NB Only certain points are used in the fits as discussed in the text.)

respectively. In general the odd ν data do not always fit the LK formula quite as well but we would expect the results to be accurate to $\pm 10\%$.

We have also measured the activation energy Δ from an Arrhenius plot of $\rho_{xx} = \rho_0 \exp(-\Delta/2kT)$ as shown in figure 3. By contrast this only uses data at the lowest temperatures. A comparison of the results from the two techniques can be seen in figure 4, which shows the energy gaps E_g deduced from the LK method and Δ from an Arrhenius plot in sample G586 for pressures in the critical range 10–20 kbar. The difference between the two values is due to the finite width Γ of the extended states caused by LL broadening. Provided the density remains unchanged this should be a constant such that $E_g = \Delta + \Gamma$. As the gap becomes small the LLs overlap, no well developed resistivity zero is observed and the activation behaviour collapses. This can be clearly seen for the 20 kbar data in figure 3. Consequently the values deduced from the Arrhenius plots become highly questionable. Above 17 kbar even the LK fits fail systematically. This may be due in part to the fact that for the two highest pressures the maximum density achieved by prolonged illumination is significantly lower than for the lower pressures. At low temperatures the minima do not become zero so $\Delta\rho$ does not reach 100%. Additionally, at the high-temperature end of our data range the resistivity shows an unusually slow temperature dependence, which would be

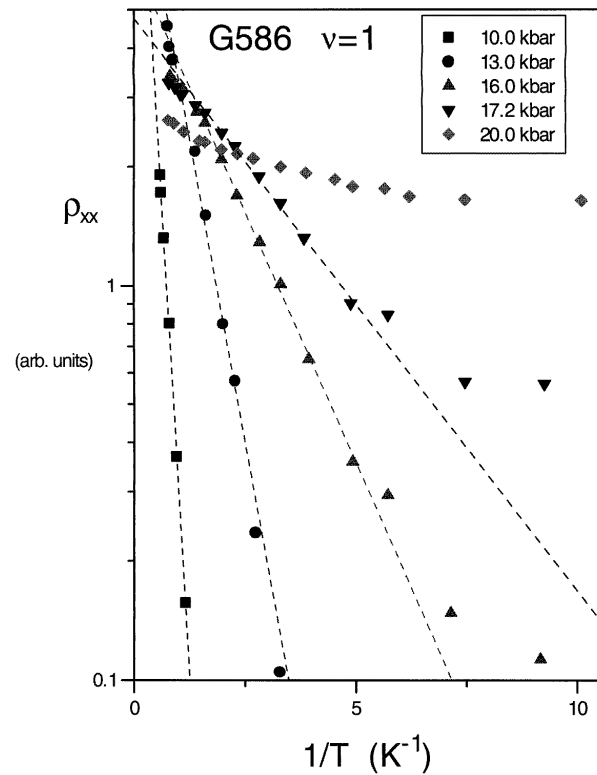


Figure 3. Arrhenius plot of the resistivity at $\nu = 1$ for sample G586, including fitted lines from which the activation energy Δ is obtained.

interpreted as a very large energy gap if the LK formula were still valid. The values of the energy gaps shown on figure 4 are for temperatures below this deviation from the $X/\sinh X$ law, but those at the highest pressures must still be regarded as relatively uncertain. Notwithstanding these qualifications, the gap at $\nu = 1$ clearly decreases as the pressure is increased. There is also some evidence from the higher temperature traces that it reaches a minimum at ~ 18 kbar and beyond this pressure the gap recovers again, although the low-temperature resistivity zero is not recovered. Before discussing the nature of the $\nu = 1$ energy gap, we note that the existence of a symmetry about 18 kbar, however limited, is good evidence that the g -factor has really passed through zero at the pressure predicted by $\mathbf{k} \cdot \mathbf{p}$ theory and has indeed changed sign at the higher pressures. Further evidence of this can be seen at $\nu = \frac{1}{3}$, where the energy gap also collapses while the resistivity minimum disappears and then reappears above 18 kbar [21].

We do not believe that the collapse of the energy gap is due to pressure adversely affecting the mobility, for three reasons. First, at a given density the zero-field mobility showed an initial increase but thereafter did not vary as the pressure was applied. Secondly, as can be seen in figure 1, strong fractional QHE features were present at low temperature for all pressures [17,21] and in particular the feature at $\nu = \frac{2}{3}$ in sample G586 had an essentially constant energy gap for pressures between 10 and 20 kbar. Finally, the single-particle lifetime τ_s has

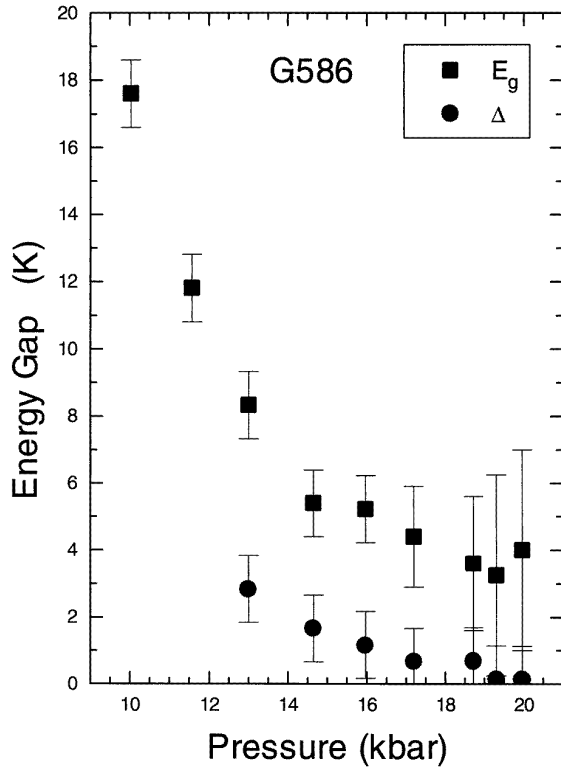


Figure 4. Energy gap at $\nu = 1$ for sample G586 for pressures between 10 and 20 kbar. Notice how the gap decreases with pressure, until 18 kbar where $g = 0$.

been obtained from a Dingle analysis of the low-field, even-integer, Shubnikov–de Haas oscillations. Although there is some uncertainty in the values of τ_s , since there are not many oscillations visible and the background resistivity is field dependent, the data above 13 kbar consistently yield a value of 1.3 ± 0.2 ps, which suggests that the scattering of spin unpolarized electrons is not affected by the pressure. Therefore we conclude that the change in energy gap at $\nu = 1$ is connected with a change in spin stiffness caused by the effects of pressure on g .

3. Spin waves at $\nu = 1$

Before deciding whether these observations provide evidence for skyrmions we will consider data taken from the two higher-density samples at lower pressures, for which skyrmion formation is less likely when $\nu = 1$ occurs at high field. Typical magnetoresistance data from sample G902 are shown in figure 5 for temperatures between 1.5 and 7 K and values of E_g for $\nu = 1$ in samples G627 and G902 are shown in figure 6 as a function of carrier density. For these samples the resistivity at $\nu = 1$ always becomes zero at sufficiently low temperature and the data fit the LK formula very well. Figure 6 demonstrates that the carrier density is a parameter that unifies the data from both samples even though many different pressures were used. The dashed curve is the best fit of $E_g \propto \sqrt{n_e}$ and since at $\nu = 1$, $l_B \propto n_e^{-0.5}$ this shows that the gap

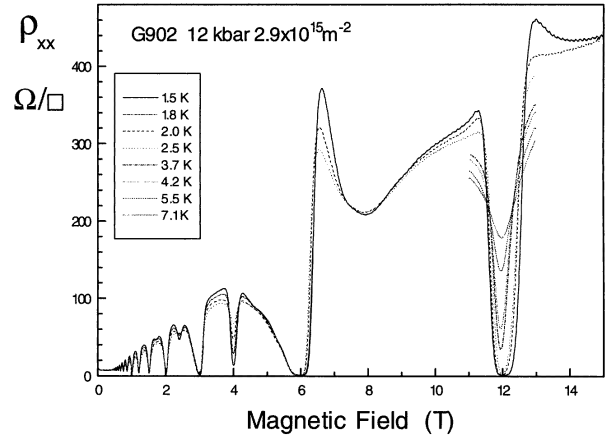


Figure 5. Magnetoresistance for sample G902 at 12 kbar showing the temperature evolution of the minima at $\nu = 1$ and 3.

is dominated by the Coulomb energy expected for spin-wave excitation. The equation of the line is $E_g = 0.22 E_c$ which gives a spin stiffness considerably smaller than the theoretical estimate of $\sqrt{\pi/2}$ ($= 1.25$) for an ideal 2DEG. Often such discrepancies between theory and experiment can be explained by the finite thickness of the real 2DEG softening the Coulomb interaction, but this usually only accounts for a factor of two [22]. Our values are generally in line with previous experiments [4] so the source of discrepancy remains an open question. The square-root dependence reported here might be thought to be at variance with the linear behaviour of [4]; however, the gaps reported by Usher *et al* [4] were activation energies (Δ) which do not include the LL broadening. Thus, although the large gaps at high density agree with our data, the smaller gaps are underestimated. The current data have also been analysed using Arrhenius plots and show Δ increasing linearly with n_e for each sample but with a sample-dependent offset that increases with disorder. The universal curve of figure 6 can only be obtained when the LL broadening is correctly accounted for in the analysis.

Also included in figure 6 are the energy gaps at $\nu = 3$ and 5, which have been plotted at the equivalent density n_e/ν^2 . The squared filling factor is required to account both for lower magnetic field and the reduced number of electrons participating in the collective motion. It is seen that the universal curve for $\nu = 1$ also describes the energy gaps at higher odd integer ν which suggests the gaps are all determined by the same mechanism of spin-wave excitation. In fact, enhancement of the spin gap at higher odd integers appears always to give the same spin stiffness provided the correlation energy is greater than the disorder potential [23, 24].

4. Skyrmions at $\nu = 1$

We now return to the search for skyrmions. In order to compare the experimental data with theory, the gaps have been scaled by the Coulomb energy and plotted as a function of η in figure 7 for all the samples. On such a plot

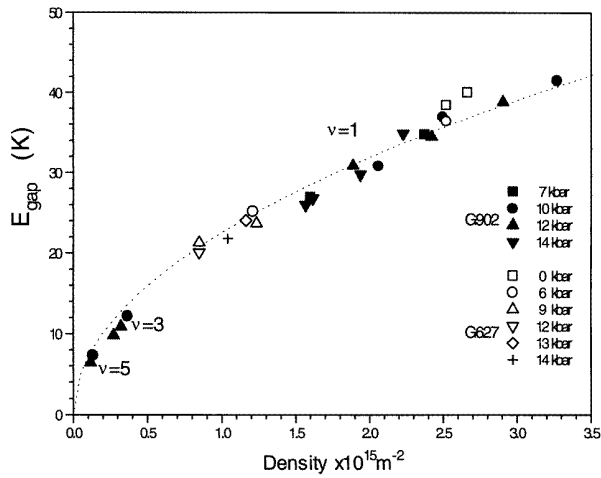


Figure 6. Energy gaps at odd-integer ν for G627 and G902. The dashed curve follows $n_g^{0.5}$ expected for spin-wave excitation. Data for $\nu = 3$ and 5 have been plotted at the effective density n_g/ν^2 .

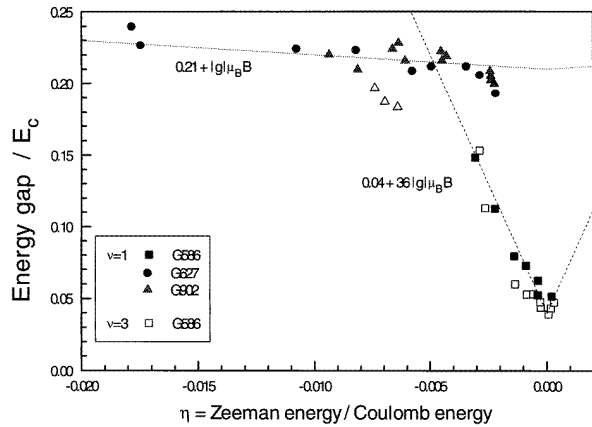


Figure 7. Energy gaps for all the samples as a function of ZE. Note that both axes are scaled by the Coulomb energy E_c . Solid points are for $\nu = 1$, open for $\nu = 3$. The dotted lines with gradients of ± 1 show the energy to create a single spin exciton. The dashed lines have gradients of ± 36 corresponding to skyrmion excitation.

an energy gap that scales only with the Coulomb energy would show up as a horizontal line and a single-particle spin gap would follow a line with unit gradient passing through the origin. For samples G902 and G627 the gap at $\nu = 1$ appears to scale with the Coulomb energy plus the much smaller SP ZE, represented by the dotted line with unit gradient. This corresponds to the simple spin wave as expected from the above discussion. By contrast the data taken above 9 kbar for G586 with $|\eta| < 0.0035$ exhibit a rapid change proportional to the ZE but with a slope much greater than unity. The dashed lines on figure 7 have gradients of ± 36 which describe the data well at small $|\eta|$. According to the arguments of [10] and [11] this suggests the energy gap has a component $36g\mu_B B$ and indicates an excitation involving the reversal of 36 spins.

Theory suggests that the skyrmion size should increase continuously as $|\eta|$ is reduced so this value of $R = 36$ should be taken as only an average or limiting value.

Kamilla, Wu and Jain estimated the number of reversed spins in an anti-skyrmion by minimizing its energy [25] $E(R)/E_c = 0.313 + 0.23 \exp(-0.25R^{0.85}) + \eta R$. This shows that an anti-skyrmion with 18 reversed spins (i.e. 36 in the pair excitation) would occur at $|\eta| = 0.0017$ which falls right in the middle of our data range, and that R falls to 11 by $|\eta| = 0.0050$. However, the minimum energy of $0.313E_c$ at $\eta = 0$, which corresponds to a pair gap of $0.627E_c$, is very much larger than the $0.04E_c$ observed experimentally.

Careful inspection of figure 7 suggests that the gap for the other samples may also be about to fall once $|\eta| < 0.003$. If this were the case the same analysis would suggest skyrmionic excitations of even larger numbers of spins. However, there is a large uncertainty in this number due to the uncertainty both in the absolute value of pressure and the precise pressure at which the ZE will be zero, which is slightly magnetic-field dependent. This uncertainty is much less for G586 due to the rapid decrease in gap which is observed, and the suggestion of a minimum energy gap which allowed us to confirm the pressure where $g = 0$.

We are led to deduce that the excitations contain these very large numbers of spins because the gap does not change until $|\eta|$ is quite small and then drops to a very small value. The experiment suggests that the minimum gap for skyrmionic excitations is $0.04E_c$ compared to $0.21E_c$ for the spin-wave gap at vanishing ZE. This is substantially different from the prediction of exactly a 50% reduction, made in [3], for infinite-sized skyrmions. However, since the experimental gap for creating a spin wave is already five times smaller than the theoretical prediction it is not surprising that quantitative agreement with the skyrmion theory is incomplete. It may be that the more fundamental question to address is why the spin-wave energy is so small or equivalently why real samples have such low spin stiffness. Another qualitative difference from the theory is that instead of the cusp which would result if $R \rightarrow \infty$ as $g \rightarrow 0$ the minimum of figure 7 is more rounded, as found in [11]. This may be explained by long-range disorder limiting the skyrmion size. At the density of $0.44 \times 10^{15} \text{ m}^{-2}$ an 18-spin skyrmion would have a radius of 1140 \AA which is already larger than the spacer layer thickness that usually determines the scale of the disorder potential in modulation-doped structures. A theoretical estimate of how the skyrmion size is limited in real systems would be very useful at this point.

5. Width of the $\nu = 1$ minima

Transport data provide another measure of the LL structure through the width of the QHE plateaux in ρ_{xy} , or equivalently the minima in ρ_{xx} . This is a measure of the number of localized states that must be passed through before conduction can take place through the extended states. For a low-mobility sample, with a large number of localized states, the plateaux become very wide at low temperature with extremely sharp risers and ρ_{xx} consists of a set of δ -functions, i.e. $\delta\nu$, the width in filling factor of the $\nu = 1$ minimum, is close to unity. In the limit of $T \rightarrow 0$

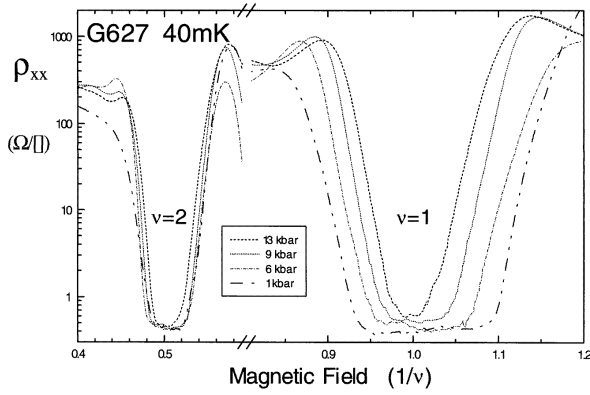


Figure 8. Resistivity as a function of inverse filling factor for sample G627 showing how the width of the $\nu = 1$ minimum decreases with increasing pressure.

there will only be one extended state. By contrast, high-mobility samples which show a lot of structure between the integer plateaux and exhibit the fractional quantum Hall effect (FQHE) would appear to have a large number of extended states at $T = 0$. In that case the plateaux are very narrow and $\delta\nu \rightarrow 0$.

Figure 8 shows how the widths of the minima at $\nu = 1$ and 2 change with pressure for sample G627 at 40 mK and a constant density. The log scale focuses attention on the low-resistivity region of ρ_{xx} which shows when conduction through extended states begins. At $\nu = 1$ $\delta\nu_1$ decreases dramatically as the pressure is increased. By contrast, at $\nu = 2$ $\delta\nu_2$ only changes by a very small amount which is related to the slight increase in mobility at higher pressure. It should also be noted that the minima are quite symmetrical about the integer filling factor, showing that quasi-electrons and quasi-holes are localized to the same degree.

We have measured $\delta\nu_1$ and $\delta\nu_2$ for each of the samples, at the lowest temperature possible, both at a fixed resistivity, in this case $10 \Omega/\text{sq}$, and at fixed fractions (1% and 10%) of $\rho_{xx}(\infty)$. As expected, the plateaux widths vary significantly between samples, due to their different respective mobilities and the criterion used to established the width. However, we find that the ratio $\delta\nu_1/\delta\nu_2$, with the same criteria used for each minimum, is much less sensitive to the temperature of the measurement or the actual value of resistivity chosen and shows a universal trend for all samples regardless of their mobility. This ratio is shown using the minima widths at $10 \Omega/\text{sq}$ in figure 9 as a function of η . (Data are not included for G586 at the highest pressures as the $\nu = 1$ minima do not approach zero closely enough, i.e. the Hall plateaux are not flat, although they clearly exist at the correct value. However, if we look higher up in the minima, at the 50% level, it appears that $\delta\nu_1/\delta\nu_2$ increases again above 18 kbar, i.e. the minima become wider again once g has changed sign.) It is quite remarkable that the ratio $\delta\nu_1/\delta\nu_2$ appears to be independent of sample-specific parameters like density or mobility, while the individual quantities $\delta\nu_1$ and $\delta\nu_2$ vary enormously. (We have also examined this ratio for a number of other structures with mobilities covering over

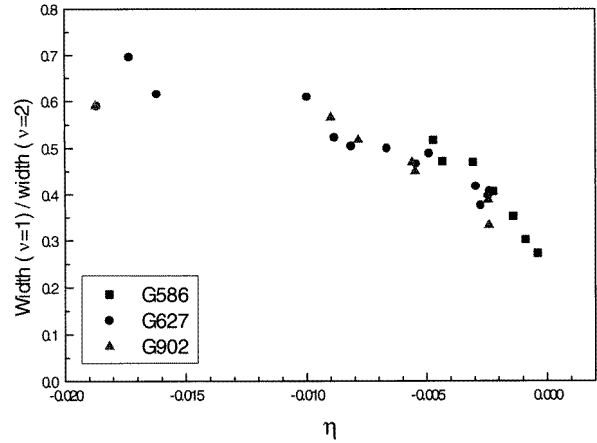


Figure 9. Ratio of the width of the minima at $\nu = 1$ to $\nu = 2$ showing a universal trend among all the samples to decrease with η .

two orders of magnitude, and at zero pressure this gives a ratio of 0.6 ± 0.1). As the ZE is varied most of the change in the ratio comes from $\delta\nu_1$ while $\delta\nu_2$ remains fairly steady for a given sample, although both widths vary with temperature in a similar way to make the ratio insensitive to temperature. If we interpret $\delta\nu_2$ as a measure of the number of localized single-particle states then we might expect $\delta\nu_1/\delta\nu_2 = 1$ when conduction at $\nu = 1$ is also by single particles. The ratio could be slightly less than unity due to the smaller gap at $\nu = 1$ compared to the magnitude of the localization potential and this would explain the experimental data at large $|\eta|$. However, when collective phenomena are dominant at $\nu = 1$, which we expect at small $|\eta|$, localization will be very different. For strong localization we would expect a pinning of the states leading to a wider localized region, but only in the region close to $\nu = 1$ where the skyrmions are formed. This is the case for a Wigner crystal where it is only necessary to pin *one* particle to lock the whole lattice into place and make the system insulating. Thus strong localization would predict that $\delta\nu_1$ should increase as $|\eta|$ decreases, contrary to the observations. For a weaker long-range localizing potential we might expect that a state will only be localized if *all* its constituent particles are localized. This means that for a fixed number of localization sites there will be more extended states for collective motion than for single-particle motion and so $\delta\nu_1$ will be less. A similar argument would explain the observation that samples with narrow plateaux show the FQHE and vice versa. Thus figure 9, where the ratio of the widths falls rapidly as η and g tend to zero, may be interpreted as showing how collective phenomena such as skyrmions become more important at smaller g -factor.

We still have to explain why $\delta\nu$ shows the same behaviour for all the samples, while E_g does not. There is a significant difference between these two measurements. In the former case the filling factor is changed, i.e. flux quanta are added to or subtracted from the system which results in an excess of holes or electrons of reversed spin, which may be dressed to become skyrmions or anti-skyrmions. In the latter case a pair excitation is created with exactly one

flux quantum per particle at $\nu = 1$. It would thus appear that there is some difference between creating a single or paired excitation.

6. Skyrmions at $\nu = 3$

We now turn to the data at $\nu = 3$. As seen in figure 6, the energy gaps at $\nu = 3$ for sample G902 are approximately consistent with the spin-wave picture. When these data are scaled by E_c at the relevant field and plotted on figure 7 it can be seen that they lie slightly below the $\nu = 1$ data and decrease somewhat with $|\eta|$. The relationship between data from $\nu = 1$ and 3 is different in the two figures since figure 6 considers the effect on the gap of the number of electrons participating whereas in figure 7 the parameter η measures the ease of skyrmion formation. Again there is a much larger effect seen in the results from G586. For this sample there is a dramatic decrease in gap between the two data points taken below 11 kbar and those at higher pressure. The pressure variation of the gaps deduced from the high-pressure data is barely larger than their uncertainty, but it should be realized that the change in ZE is much smaller than over the same pressure range for $\nu = 1$. When the data are correctly scaled and plotted on figure 7 they show a remarkable similarity to the data from $\nu = 1$. This strongly suggests that the same mechanism is responsible for the excitations at both of these filling factors. Hence we are led to conclude that skyrmionic excitations also occur at $\nu = 3$ near to $g = 0$.

Contrary to the ideal case [12], this is indeed possible because the finite thickness z of the 2DEG has the effect of softening the Coulomb interaction once $z \sim l_B$ [14]. For a density of $5 \times 10^{14} \text{ m}^{-2}$ the magnetic length at $\nu = 3$ is 213 Å and, using the variational method for a triangular well [26], the mean distance of electrons from the interface is 219 Å. According to [14] with $z = 2l_B$ skyrmions will be excited at $\nu = 3$ provided $|\eta| < 0.0044$, which covers the whole range of the G586 data. It also predicts that at $|\eta| = 0.002$, $R = 15$ and $E_g = 0.48E_c$. The first prediction is quite consistent with our data as a line drawn to include all the $\nu = 3$ points on figure 7 would have a gradient of ~ 22 , but the size of the measured energy gaps is again an order of magnitude smaller than the theoretical predictions.

7. Role of disorder

Finally it is necessary to consider whether these data really do provide evidence for large skyrmions in the 2DEG of high-mobility samples or if there is an alternative explanation for the precipitous drop in energy gap as $g = 0$ is approached. It has been suggested that disorder may play an important role in determining the spin stiffness of the system [23, 27]. If the disorder potential is smaller than the ZE, it will play no significant role and spin waves will be created as normal. However, once the disorder potential is comparable to the ZE, reversed spins will already exist in the ground state. This reduction in spin stiffness makes it easier to perform additional spin flips so the spin waves become dressed, which in turn reduces the spin stiffness further. The excitations at very small ZE will thus

contain many reversed spins. What is not clear is whether this mechanism will lead to a skyrmionic spin texture or merely a multiple spin exciton. If the transition from single reversed spin excitations to multiple reversed spin excitations is critically driven by disorder then the energy gap may decrease more rapidly as $g = 0$ is approached than was the case in the variable-sized skyrmion model discussed above, where there is a smooth change in size of the collective excitation. (We note that with the disorder-driven picture it would not be possible to infer the numbers of spins involved in the excitations, either in this work or [10] and [11].) So does this model of disorder-induced skyrmion formation fit with our observations? Figure 7 shows that the drop in energy gap begins at larger $|\eta|$ for G586 than for G627 or G902 but this is largely due to $\nu = 1$ being at lower magnetic field. In fact the ZE where the drop begins is ~ 0.3 K in each case, which would be expected as the samples are all fairly similar. However, this is a very small value for a disorder potential when compared with typical LL widths of several kelvin. Furthermore it is far from clear why the disorder could have such a dramatic effect on the exchange energy at $\nu = 1$ without destroying the correlations responsible for the FQHE at $\nu = \frac{2}{3}$. It would have to be a strange type of disorder to affect the spin system without upsetting the spatial correlations. Finally, it is again hard to see how the minimum energy gap could be smaller than for an infinite-sized skyrmion–anti-skyrmion pair. Thus the disorder-based explanation raises at least as many problems as it solves and at present there are no detailed theories with which to make comparisons. We therefore suggest that the data have a more convincing explanation in terms of skyrmions at vanishing ZE.

Another possibility that we should consider is the phase separation at $g = 0$ of a spin polarized $\nu = 1$ system into an unpolarized system of two half-filled LLs, similar to that observed in bilayers where the phases correspond to either $\nu = 1$ in a single layer or $\nu = \frac{1}{2}$ in two layers. The decrease in gap and the strange temperature dependence of the highest pressure data might then be explained as a transition from a one-component to a two-component phase. There are a number of reasons why we dismiss this possibility. First, we always observe a quantized Hall plateau which would not be expected for states based on $\nu = \frac{1}{2}$. Additionally, in the bilayer system the two-component phase is destroyed when the layer separation is small and for our situation there is no physical separation of the spin-up and spin-down electrons. The strong ferromagnetic interactions would then prevent any phase separation. However, the possibility remains of forming spatially separated spin-up and spin-down domains with the cost of forming domain boundaries being paid by the disorder potential. This would certainly limit the size of any skyrmions and account for the rounded minimum in figure 7, but causes problems in producing well separated skyrmion–anti-skyrmion pairs.

8. Conclusion

In summary we have measured the energy gaps for the ferromagnetic states at $\nu = 1$ and 3 under conditions

where the ZE can be tuned through zero. At large ZE the excitations involve a single reversed spin and are the well known spin waves. As the ZE is reduced to zero by applying hydrostatic pressure, the energy gap decreases dramatically. At small ZE the excitations appear to consist of a large number of reversed spins which can be interpreted as skyrmion–anti-skyrmion pairs. The same behaviour is seen both at $\nu = 1$ and $\nu = 3$ which suggests that the finite thickness of real 2DEGs makes skyrmions the lowest-lying excitations not only at $\nu = 1$ but also at other odd filling factors.

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