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The dependence of the Composite Fermion effective mass on carrier density and Zeeman energy

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Abstract. Measurements of the temperature-dependent resistivity of high-mobility GaAs/GaAlAs heterojunctions are used to measure the effective mass of Composite Fermions (CF). The CF effective mass is found to increase approximately linearly with the effective field B^* up to effective fields of 14 T. Data from all fractions around $\nu = 1/2$ are unified by the single parameter B^* for samples studied over a wide range of temperature. The energy gap is found to increase as $\sqrt{B^*}$ at high fields. Hydrostatic pressure is used to reduce the value of the electron *g*-factor, and this is shown to have a large effect on the relative strengths of different fractions. By 13.4 kbar, where the Zeeman energy is only 1/4 of its value at 0 bar, fractions with odd numerators are found to be strongly suppressed, and new features with even numerators appear. The energy gaps measured for 5/3 as a function of carrier density and pressure are consistent with a *g*-factor equal to the bulk value enhanced by a factor of two due to exchange interactions.

The fractional quantum Hall effect has been known for many years [1, 2], but our picture of the phenomenon is changing rapidly due to the introduction of the Composite Fermion (CF) model [3, 4]. In this model the Coulomb interaction of one electron with all the others is replaced with a Chern–Simons gauge field, equivalent to attaching an even number ($2m$) of flux quanta ($\Phi_0 = h/e$) to each electron. In a mean field approximation the gauge field exactly balances the external field at filling factor $\nu = 1/2m$ where the system of interacting electrons in high magnetic field is replaced by one of independent CFs in zero field. At other filling factors there are more (or fewer) flux quanta than required to cancel the gauge field and the CFs see an effective magnetic field, $B^* = B - 2m\Phi_0 n_e$. This leads to quantization of the CF energy into Landau levels (LLs) and gaps open in exact analogy with the integer quantum Hall effect (IQHE) of non-interacting electrons. Thus the FQHE (fractional QHE) may be simply regarded as the IQHE of composite fermions. This behaviour is illustrated in figure 1, which shows the resistivity of a high-mobility GaAs/GaAlAs heterojunction as a function of the magnetic field, written in terms of the density of flux quanta per electron. In the region $0 < B/n_e < 1$ we see the Shubnikov–de Haas oscillations of the single-particle electrons, while from $1 < B/n_e < 3$, corresponding to $-1 < B^*/n_e < 1$, we see the Shubnikov–de Haas oscillations of the first generation ($m = 1$) composite fermions. From $3 < B/n_e < 4$, corresponding to $-1 < B^*/n_e < 0$, we see second-generation ($m = 2$) oscillations. This picture becomes more complex in the regions of $0.5 < B, B^*/n_e < 1$ when regarded in detail, since further features appear at fractions such as 4/3 and 5/3 and their higher-generation analogues 5/7 and 4/5. In this case mixed states appear which consist of filled levels of more than one type of particle, such as single-particle electrons in a filled Landau level and CF states formed from the remaining electrons.

By treating the FQHE features in ρ_{xx} as CF Shubnikov–de Haas oscillations, it is possible to deduce an effective mass of the CF particles [5–8] by analysing the temperature dependence of the oscillation amplitudes [9]. Work on low-density samples suggested that the effective mass M^* could be described by the relation $M^* = 0.51 + (0.074)B^*$, in units of the free electron mass m_e. The same dependence was found for positive and negative values of B^*, and the mass values from samples with n_e differing by a factor of two was found to be the same at a given value of B^*. There have also been reports that the mass may diverge as

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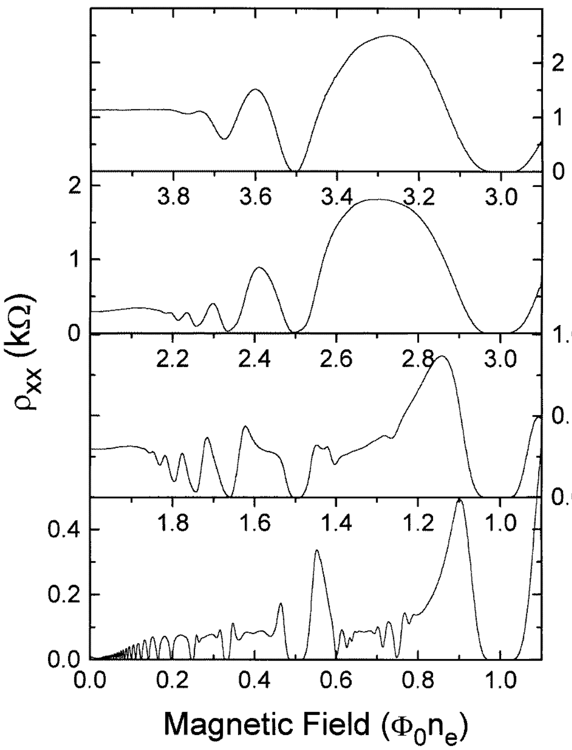

Figure 1. A plot of the FQHE features in a GaAs/AlGaAs heterojunction with a carrier density of 9×10^{10} cm^{-2} measured at 97 mK. The data have been replotted as a function of magnetic field to emphasize the symmetry of the oscillations.

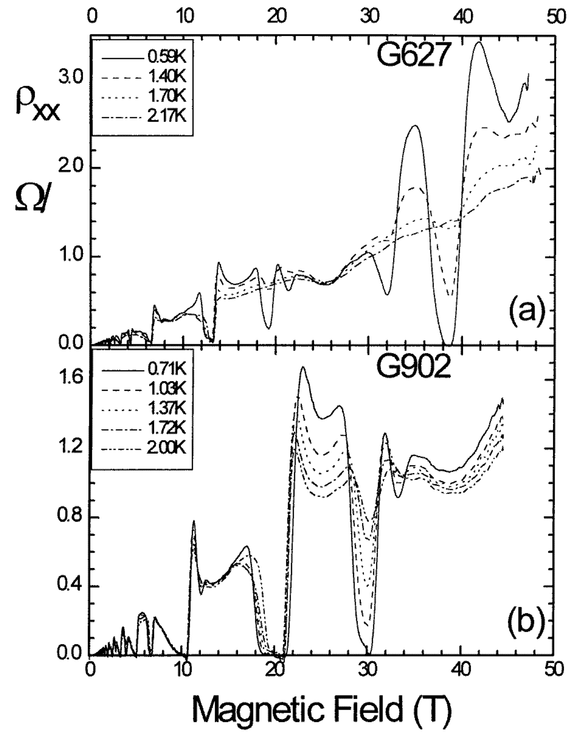

Figure 2. Typical experimental traces taken in two GaAs/AlGaAs heterojunctions with carrier densities of 3.0 and 4.8×10^{11} cm^{-2} measured in a pulsed magnetic field as a function of temperature.

$\nu = 1/2$ is approached [6–8], where the oscillations in ρ_{xx} are weak. Studies of the mass have recently been extended to much higher densities by the use of pulsed magnetic fields up to 50 T, combined with ^{3}He temperatures [10]. In these measurements good thermal equilibrium was ensured by immersing the sample in liquid, and using low measuring currents and keeping induced voltages to a minimum during the 10 ms duration pulses. Some typical experimental recordings are shown in figure 2, for samples with carrier densities of 3.2 and 4.8×10^{11} cm^{-2}. The oscillations in resistivity are analysed using the Ando formula [9].

$$\frac{\Delta\rho_{xx}}{\rho_0} \propto \frac{X}{\sinh X} \exp\left(-\frac{\pi}{\omega_c \tau_q}\right) \cos 2\pi(\nu - \tfrac{1}{2}) \qquad (1)$$

where $X = 2\pi^2 k_B T/\hbar\omega_c$ and $\omega_c = eB/m^*$ is the cyclotron frequency. For composite fermions we replace B by B^*, ν by ν^*, m^* by M^* and τ_q by T_q. The analysis was performed on the features that are large at low temperature for the range of conditions corresponding to $\Delta\rho/\rho < 0.5$, where the oscillations are only a weak modulation of the conductivity and the higher harmonics can be neglected.

The results of the analysis are shown in figure 3 as a function of n_e for several different fractions. There is no unique n_e dependence covering all fractions, but it can clearly be seen that instead the mass values fall in pairs, corresponding to states with a common numerator p, for example 2/3 and 2/5. These have equal numbers of occupied CF Landau levels, but occur on either side of $\nu = 1/2$ with effective fields in opposite senses. This provides a simple demonstration of the symmetry of the states about $\nu = 1/2$ which is consistent with the CF model, rather than that of particle–hole conjugation where states of common denominator q (for example 1/3 and 2/3) look similar. In the low-density limit all states tend to the same effective mass, but the lower-index CF Landau levels show an increasing 'non-parabolicity'. By $n_e = 3 \times 10^{11}$ cm^{-2} the effective masses for 1/3 and 2/3 differ by approximately 40%. In addition, the gradients of each line in figure 3 show an accurate ($\pm$ 2%) $1/p$ dependence. The CF mass may therefore be described by the expression

$$M^* = 0.51 + \frac{0.35}{p} n_e = 0.51 + 0.083 B^* \qquad (2)$$

in units of m_e, with n_e in units of 10^{11} cm^{-2}.

The dependence of M^* on B^* is shown in figure 4(a) for a wide range of samples, which shows that there is a simple functional dependence of M^* on the single parameter B^*, covering moe than a factor of 25 variation in B^*. The measurement of M^* is equally a measurement of the CF cyclotron energy $E_c^* = \hbar e B^*/M^*$. This E_c^* is equivalent to what would previously have been known as the FQHE energy gap Δ, for a sample with infinitely narrow levels. In figure 4(b) we show E_c^* as a function of B^* for all of the samples studied. The increase of M^* with field necessarily means that E_c^* shows a sublinear increase with effective field. The broken curve in figure 4(b) is a fit to the data with $E_c^* = a\sqrt{B^*}$, where $a = 3.3$ K T$^{-1/2}$.

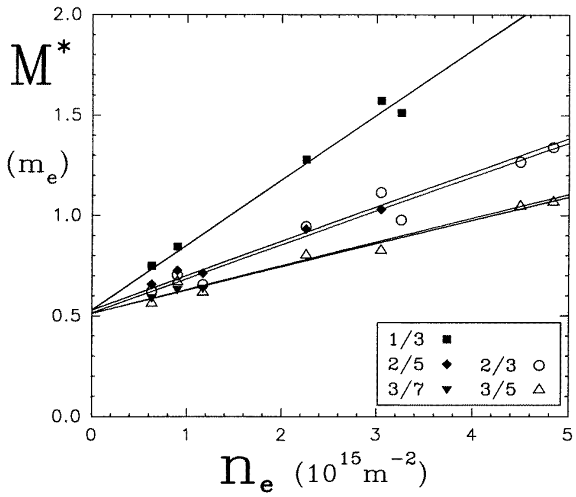

Figure 3. The CF effective mass as a function of carrier density for different fractions. The full lines show least squares fits.

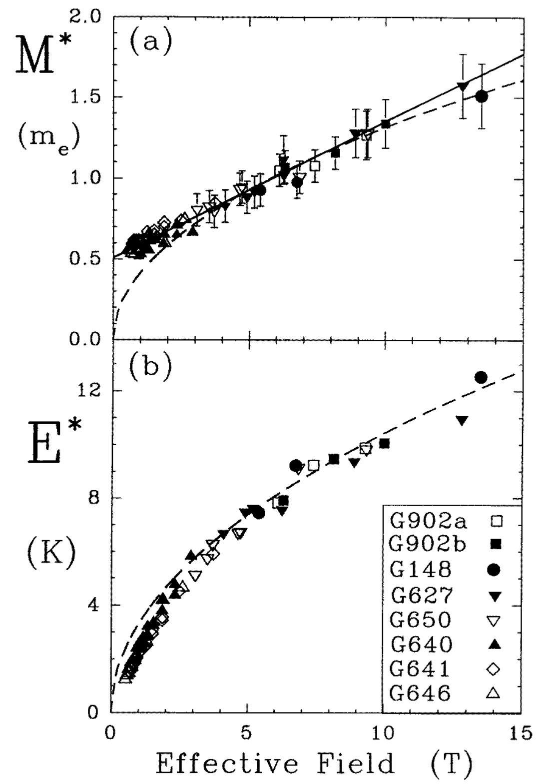

Figure 4. (*a*) The effective mass as a function of effective field B^* for the family of fractions around $\nu = 1/2$ for a range of different samples with different carrier densities. (*b*) The CF effective gaps deduced from the effective masses as a function of effective field for the same samples as in (*a*). The full and broken curves are discussed in the text.

Although this is not the function generated by a mass that increases linearly from an offset zero, above a value of $B^* = 1$ T there is remarkably good agreement with this single dependence on B^* alone.

The FQHE is the result of a many-body Coulomb interaction, and so theoretically we would expect the energy gaps to scale with a relation of the form $\Delta = C_\nu e^2/(4\pi\varepsilon\varepsilon_0 l_0)$, where l_0 is the cyclotron radius (proportional to the interparticle spacing) and C_ν is a fixed coefficient, different for each fraction. Halperin *et al* [4] have used this relationship and argued on dimensional grounds that the high-field limit of M^* should show a $\sqrt{B}$ dependence with changing carrier density through l_0. This corresponds to a $\sqrt{B}$ dependence for both E_c^* and M^* for any given fraction, but our results suggest that there is not a single functional dependence on n_e but instead on B^*.

For fractions at values of occupancy greater than 1 the picture is more complex, since both spin states of the lowest single-particle Landau level must have some finite occupancy. It has long been known that increasing the value of the spin splitting by tilting the sample relative to the applied magnetic field can influence the relative strength of the FQHE states [11–13]. Recently Du *et al* [14] have shown that for fractions greater than 1 it is possible to analyse the CF states as the oscillations due to CF *holes* with a concentration $N_h^* = 2B/\Phi_0 - n_e$ in an effective field $B^* = 2n_e\Phi_0 - 3B$. In this picture the 5/3 state consists of a single occupied hole CF Landau level, with its gap determined by the smaller of E_c^* and the Zeeman energy $g^*\mu_B B$, while 4/3 has two occupied levels, which for the untilted case represents an unpolarized state with one level from each spin. On increasing the Zeeman energy by tilting, the two families of CF Landau levels move through each other causing features with odd and even numbers of filled CF levels to oscillate in strength. This led them to deduce a value for g^* of $0.61 + 0.087B^*$ for one density.

In this paper we have varied both the carrier density and studied the effects of *decreasing* the Zeeman energy by the use of hydrostatic pressure. Applying pressure decreases the g-factor for electrons via the well known $\boldsymbol{k}\cdot\boldsymbol{p}$ perturbation theory expression [15]

$$g^*/2 = \frac{P^2}{3}\left(\frac{1}{E_g} - \frac{1}{E_g + \Delta_0}\right) - 1 \qquad (3)$$

where the increase in bandgap due to the application of pressure causes the g-factor to pass through zero in the region of 17 kbar for GaAs. Figure 5 shows the resistivity for a sample with a zero-pressure carrier density of 3×10^{11} cm^{-2} for four different pressures from 0 to 13.4 kbar. As the pressure increases, the maximum achievable carrier density falls, due to the decrease of the band offset difference between the GaAs and the AlGaAs barrier, and by 13.4 kbar has almost halved. The curves are shown normalized to the level occupancy. Several features are apparent. First, the fractions with odd values of the numerator show a decrease in strength which is particularly pronounced in the region $1 < \nu < 2$. For example the states at 5/3 and 7/5 almost completely disappear by 13.4 kbar. Secondly, there is an enhancement for most of the even-numerator states, and new states appear such as 4/5 and 6/5. Finally there is a decrease in the width of the resistivity minimum at $\nu = 1$. This behaviour illustrates the important role played by the Zeeman energy in determining the behaviour of the different states. By

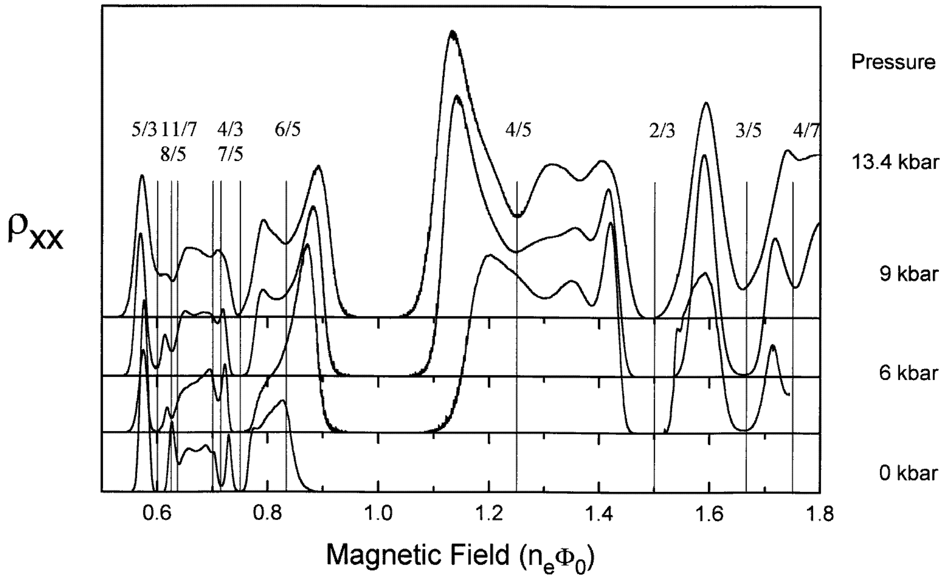

Figure 5. The resistivity as a function of normalized magnetic field for a GaAs/AlGaAs heterojunction with a carrier density of 3.0×10^{11} cm^{-2} at zero pressure, which falls to 2.1, 1.87 and 1.6×10^{11} cm^{-2} at pressures of 6, 9 and 13.4 kbar.

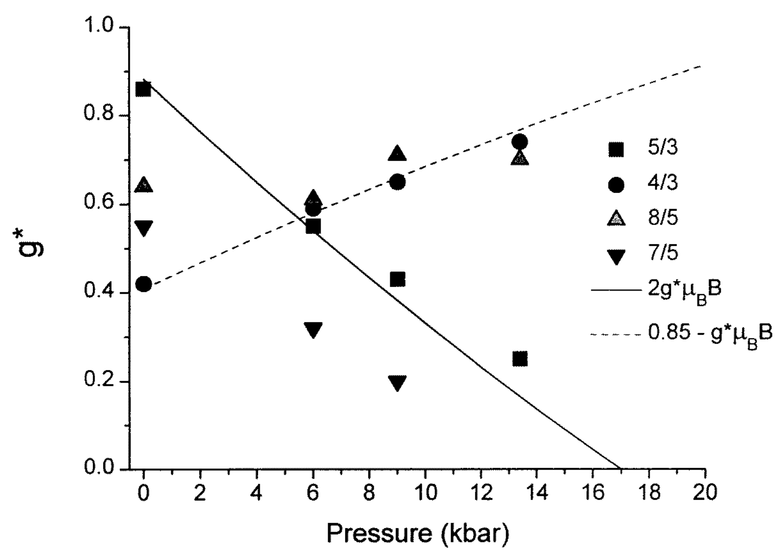

Figure 6. The effective g-factor as a function of pressure, deduced by putting the CF energy gap $E_c^* = g^*\mu_B B$, where B is the total magnetic field.

13.4 kbar we estimate using equation (3) that the GaAs g-factor has fallen to a value of order 0.1 or less, which means that for the sample studied here the Zeeman energy is only of order 0.3 K in the region of $\nu = 3/2$, much less than the activation energies of all of the fractions studied. Under these conditions it appears as if the system is behaving as doubly degenerate 2D layer, with strong fractions only given by two times the usual occupancies.

To analyse the pressure dependence of the energy gaps, we use the same level scheme as proposed by Du *et al* [14], which indicates that 5/3 corresponds to the Zeeman energy gap between the last two CF landau levels of the two spin states with a polarized ground state, while 4/3 is an unpolarized level with an energy gap determined by the difference of the CF cyclotron energy and the Zeeman energy. Measuring the gaps by fitting the temperature dependence of the oscillation minima as described above allows us to define an effective g-factor for the system from $g^*\mu_B B = E_c^*$, where it should now be remembered that we are using the total magnetic field, and not B^*. The results of such an analysis are shown in figure 6 for several of the minima around $\nu = 3/2$. The effective g-factor measured from 5/3 shows a steady decrease with pressure, following quite closely the calculated decrease in the single-particle value for GaAs, but assuming an exchange enhancement [16] by a factor of two due to the polarization of the system.

By contrast 4/3 shows a steady increase [17], due to the fall in Zeeman energy, and since this state is unpolarized the increase follows the calculated values using the bare value of the bulk g-factor. Further confirmation of this picture comes from a study of the carrier concentration dependence of the magnitude of the 5/3 gap [18], which increases almost linearly with carrier density, and thus total magnetic field, consistent with a constant value of the g-factor of 0.85.

Returning to the minimum at $\nu = 1$, we see that there is a steady decrease in width with pressure, but the rate of fall is considerably less than the decrease in the magnitude of the calculated Zeeman energy. This behaviour is probably related to the formation of a residual Coulomb gap even for zero g-factor, as found in double-layer systems [19].

These preliminary measurements indicate that a full understanding of the role of the Zeeman energy in determining the CF properties will be vital.

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