The required problems are Exercises 8.2, 8.3, and 8.6. Please let me know if any of the problems are unclear or have typos.

This example sheet requires the following definitions. A *graph* G is a one-dimensional CW complex. A graph G is a *tree* if it is contractible. A graph G is *tree-like* if it is connected and contains no embedded circle. A 0–cell (*vertex*) of a graph is called a *leaf* if it is endpoint of exactly one end of one 1–cell (*edge*). A subcomplex $T \subset G$ of a graph is a *maximal tree* if T is a tree and T contains G^0.

Exercise 8.1. Suppose that G is a connected graph. Show that G is path-connected. Deduce that for any vertices $x, y \in G$ there is a finite embedded edge-path connecting x to y.

Exercise 8.2. Suppose that G is a graph and $L \subset G$ is an embedded circle. Show that G retracts to L. Now show that trees are tree-like.

Exercise 8.3.

- Suppose that T is a finite tree-like graph. Show that either T is a single point or T has a leaf.
- Suppose that T is a tree-like graph. Fix a pair of distinct vertices $x, y \in T$. Show that there is a unique embedded edge-path (necessarily finite) connecting x to y. This edge-path is denoted by $[x, y] \subset T$.
- [Medium.] Show that tree-like graphs are trees.

Exercise 8.4. [Medium.] Suppose that G is a connected graph. Show that G contains a maximal tree.

Exercise 8.5. Suppose that G is a connected graph and $T \subset G$ is a maximal tree where $G - T$ consists of a single edge e. Show that $\pi_1(G) \cong \mathbb{Z}$.

Exercise 8.6. Suppose that G is a connected graph. Show that $\pi_1(G)$ is a free product of copies of $\mathbb{Z}$. Now compute the fundamental group of each of the graphs below.