Please let me know if any of the problems are unclear or have typos.

**Exercise 11.1.** Suppose that $X$ is a topological space. Show that $X \times X$ is not homeomorphic to $S^1$. [Harder: do the same for $S^2$.]

We use the following notations for Exercises 11.2, 11.3, and 11.5. Suppose that $X$ is a path-connected CW complex. Fix $x_0 \in X^0$. Suppose that the set $\widetilde{X}$, the point $\widetilde{x}_0 \in \widetilde{X}$, the function $p \colon (\widetilde{X}, \widetilde{x}_0) \to (X, x_0)$, the basis $\mathcal{U}$ (for $X$), and the subsets $U_\gamma \subset \widetilde{X}$ are all as defined in lecture.

**Exercise 11.2.** Prove that

$$\widetilde{\mathcal{U}} = \{U_\gamma \mid U \in \mathcal{U}, [\gamma] \in \widetilde{X}\}$$

is a basis for a topology on $\widetilde{X}$.

**Exercise 11.3.** [Medium.] Prove that $\widetilde{X}$, equipped with the above topology, is path connected.

**Exercise 11.4.** A cover $q \colon Y \to X$ is *normal* if for every $x \in X$ the deck group $\mathrm{Deck}(q)$ acts transitively on $q^{-1}(x)$. Determine which degree-three covers of $S^1 \vee S^1$ are normal.

**Exercise 11.5.**

- Show that $\mathrm{Deck}(p) \cong \pi_1(X, x_0)$. Show that $p$ is a normal cover.
- Suppose that $q \colon (Y, y_0) \to (X, x_0)$ is a pointed, connected cover. Find a pointed map $r \colon (\widetilde{X}, \widetilde{x}_0) \to (Y, y_0)$ so that $r$ is a pointed covering and $p = q \circ r$.

**Exercise 11.6.** [Hard.] Suppose that $X$, $Y$, and $Z$ are topological spaces. Suppose that $q \colon Y \to X$ and $p \colon Z \to X$ are covering maps and suppose that $r \colon Z \to Y$ is a map with $p = q \circ r$. Must $r$ be a covering map? Prove this or give a counterexample. [Hint: see exercise 16, page 80, of Hatcher. Be aware of differences in notation.]