The required problems are Exercises 8.1, 8.2, and 8.3. Please let me know if any of the problems are unclear or have typos.

**Exercise 8.1.** Suppose that $B, C \in M_3(\mathbb{R})$ are three-by-three matrices over $\mathbb{R}$. Suppose that there is an open set $U \subset \mathbb{R}^3$ so that for all vectors $u, v \in U$ we have $u^T B v = u^T C v$. Prove that $B = C$.

**Exercise 8.2.** Set $a_\pm = (1, \pm 1, 0)$ and $b_\pm = (1, 0, \pm 1)$. Thus $M = \mathbb{H}^2 \cap \operatorname{span}(a_+, b_+)$ and $N = \mathbb{H}^2 \cap \operatorname{span}(a_-, b_-)$ are hyperbolic lines. Show that $M \cap N = \emptyset$. Find points $P \in M$ and $Q \in N$ so that the hyperbolic line $L = \overline{PQ}$ is orthogonal to both $M$ and $N$.

**Exercise 8.3.** In lecture, we defined the following functions from $\mathbb{R}$ to $M_3(\mathbb{R})$.

$$\operatorname{Rot}_\theta = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta) & -\sin(\theta) \\ 0 & \sin(\theta) & \cos(\theta) \end{pmatrix} \quad \operatorname{Tran}_t = \begin{pmatrix} \cosh(t) & \sinh(t) & 0 \\ \sinh(t) & \cosh(t) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\operatorname{Para}_x = \begin{pmatrix} 1 + x^2/2 & -x^2/2 & x \\ x^2/2 & 1 - x^2/2 & x \\ x & -x & 1 \end{pmatrix}$$

Verify that $\operatorname{Rot}_\theta \in O^+(1,2)$ and that $\operatorname{Rot}_{\alpha+\beta} = \operatorname{Rot}_\alpha \operatorname{Rot}_\beta$. Now do the same for Tran and Para.

**Exercise 8.4.** Set $L(t) = (\cosh(t), \sinh(t), 0)$. Set $A = \operatorname{Para}_1$, using the notation of Exercise 8.3. Let $M(t) = A(L(t))$. That is, $M$ is the image of the line $L$ under the hyperbolic isometry $A$. Show that $L$ and $M$ are disjoint. Now compute the distance $d_\mathbb{H}(L(t), M(t))$ for all $t$. How does the distance behave as $t$ tends to negative infinity? To positive infinity?

**Exercise 8.5.** Suppose that $S \in \mathbb{L}^3$ is a space-like vector of length one. Define $L = S^\perp \cap \mathbb{H}^2$. Define $\operatorname{Refl}_L(P) = P - 2(P \circ S)S$.

- Show that $\operatorname{Refl}_L(\operatorname{Refl}_L(P)) = P$ for all $P \in \mathbb{H}^2$.
- Show that $\operatorname{Refl}_L(Q) = Q$ for all $Q \in L$.
- Suppose that $P \in \mathbb{H}^2$ does not lie on $L$. Show that the line through $P$ and $Q = \operatorname{Refl}_L(P)$ is orthogonal to $L$.
- Suppose that $L$ and $M$ are hyperbolic lines that intersect in $\mathbb{H}^2$. Describe the composition of $\operatorname{Refl}_L$ with $\operatorname{Refl}_M$.