

The required problems are Exercise 6.3, the first part of Exercise 6.4, and Exercise 6.5. Please let me know if any of the problems are unclear or have typos.

Exercise 6.1. [Medium.] For the purposes of this problem we assume that the surface of the Earth is a perfect sphere, with radius 6367 km. Consider the three cities Providence, USA; Quebec, Canada; and Reno, USA. According to Google these are located at $(41.82^\circ\text{N}, 71.42^\circ\text{W})$; $(46.82^\circ\text{N}, 71.22^\circ\text{W})$; and $(39.53^\circ\text{N}, 119.82^\circ\text{W})$ respectively.

- For each pair of cities, compute the great circle distance between them.
- For each triple of cities, compute the angle formed by the first and third as viewed from the second.

As a sanity check, Google says that Greenwich Observatory is at $(51.48^\circ\text{N}, 0^\circ\text{W})$. The distance between Greenwich and Providence is approximately 5329 km according to Google, 5340 km according to Wolfram Alpha, and 5330 km according to my computer program. (Suggestion: write your own code to do all calculations and check your answers against on-line sources.)

Exercise 6.2. [Hard.] Suppose that $\Omega \subset S^2$ is a [spherical polygon](#) bounded by n arcs of great circles. Give a formula for the area of Ω in terms of its internal angles $\{\alpha_i\}_{i=1}^n$. Carefully justify all steps of your argument.

Exercise 6.3. Suppose $\Delta \subset S^2$ is a spherical triangle. We call Δ *equiangular* if the three angles of Δ are equal. We call Δ *Platonic* if copies of Δ tile the sphere (that is, vertices only meet vertices, edges only meet edges, and interiors are disjoint). Find all equiangular Platonic triangles and prove your list is complete.

Exercise 6.4.

- Describe all of the isometries of \mathbb{E}^2 that can be obtained as the composition of exactly two reflections. Briefly discuss *uniqueness*: that is, if T is a composition of a pair of reflections, then in how many ways is T a composition of a pair of reflections?
- Do the same for isometries of S^2 . Which ones are obtained as a composition of a pair of reflections, and in how many ways?

Exercise 6.5.

- Suppose that PQR is a spherical triangle. Compute the angles and sidelengths of the *dual triangle* which has vertices at $P^* = Q \times R / |Q \times R|$, $Q^* = R \times P / |R \times P|$, and $R^* = P \times Q / |P \times Q|$.
- Prove the second cosine law for the triangle PQR .