The required problems are Exercises 5.1, 5.2, and 5.3. Please let me know if any of the problems are unclear or have typos.

Exercise 5.1. [Exercise 2.6, of Reid-Szendrői.] The *half-turn* $\mathrm{Half}_P \colon \mathbb{E}^2 \to \mathbb{E}^2$ is the rotation, fixing $P \in \mathbb{E}^2$, through an angle of π. Prove the following.

- The composition of two half-turns is a translation.
- Every translation is the composition of two half-turns.
- The composition of three half-turns is a half-turn.
- If L is a line and P is a point then Refl_L and Half_P commute if and only if P lies in L.

Exercise 5.2. Suppose that $A \in O(n+1)$ is an orthogonal matrix. We may consider A as a function from $\mathbb{R}^{n+1}$ to itself; thus we can define $A|S^n$ to be the restriction of A to S^n. Show that $A|S^n$ is an isometry of S^n, equipped with the spherical metric.

Exercise 5.3. Suppose that $P, Q \in S^n$.

- Define the *chordal metric* d_C on S^n via $d_C(P,Q) = |P - Q|$. Show that d_C is in fact a metric on S^n.
- Show, via direct computation, that

$$2 \arcsin \left(\frac{d_C(P,Q)}{2} \right) = d_S(P,Q).$$

Exercise 5.4. [A version of Exercise 3.3, of Reid-Szendrői.] Suppose that p and q are distinct points in the metric space X. Define $B(p,q) = \{x \in X \mid d_X(x,p) = d_X(x,q)\}$. This is the set of points *equidistant* from p and q. Show the following.

- If $X = \mathbb{R}^2$ with the usual metric, then $B(p,q)$ is a line.
- If $X = S^2$ with the usual metric, then $B(p,q)$ is a great circle.

Exercise 5.5. [A version of Exercise 3.6, of Reid-Szendrői.] Suppose that ΔPQR is a spherical triangle. Let A, B, C be the sidelengths opposite P, Q, R respectively. Let α, β, γ be the internal angles adjacent to P, Q, R respectively. Prove the spherical sine law:

$$\frac{\sin \alpha}{\sin A} = \frac{\sin \beta}{\sin B} = \frac{\sin \gamma}{\sin C}.$$