The required problems are Exercises 1.1, 1.2, and 1.4. Please let me know if any of the problems are unclear or have typos.

Exercise 1.1. Suppose that $a \in \mathbb{R}$ is a scalar and $u, v \in \mathbb{R}^n$ are vectors.

- Show that $u \cdot v = v \cdot u$. [Dot product is "commutative".]
- Show that $|a \cdot u| = |a| \cdot |u|$. [Norm is linear with respect to scalars.]

Exercise 1.2. Suppose that $x, y, z \in \mathbb{E}^n$ are points and suppose that y is a barycentric combination of x and z. Now prove that $d_E(x, z) = d_E(x, y) + d_E(y, z)$.

Exercise 1.3. (See also problem A.2 in [RS].) Fix $n \in \mathbb{Z}$ a positive integer. Let $H = \{0, 1\}^n$ be the set of binary strings of length n. For strings $x, y \in H$ define the *Hamming distance* $d_H(x, y)$ to be the number of positions i so that $x_i \neq y_i$. For example, when $n = 4$ the Hamming distance between 0101 and 0011 is two. Prove $\mathbb{H} = (H, d_H)$ is a metric space. Also, compute the *diameter* of $\mathbb{H}$: the largest possible distance between a pair of points of $\mathbb{H}$.

Exercise 1.4. (Problem A.1 of [RS].) Suppose $\mathbb{X} = (X, d_X)$ and $\mathbb{Y} = (Y, d_Y)$ are metric spaces. We call a function $f \colon X \to Y$ an *isometric embedding* if for all $x, y \in X$ we have $d_Y(f(x), f(y)) = d_X(x, y)$. Justify the terminology by proving f is injective.

Show, by means of an example, that f need not be surjective even when $\mathbb{X} = \mathbb{Y}$.

Exercise 1.5. Recall the statement of the Cauchy-Schwarz inequality: For any $u, v \in \mathbb{R}^n$ we have $|u \cdot v| \leq |u||v|$. Furthermore, equality holds if and only if u and v are linearly dependent.

Here is a sketch of Schwarz's proof of the Cauchy-Schwarz inequality. Consider the function $f \colon \mathbb{R} \to \mathbb{R}$ defined by $f(t) = |u + tv|^2$. Thus f is non-negative, and is a quadratic polynomial. Thus the discriminant of f is non-positive. Finally, f vanishes if and only if u is a multiple of v.

Fill in the missing details.