Please let me know if any of the problems are unclear or have typos. Please turn in a *single* worked exercise. Record your name, the date, and the problem solved. Please also write the names of people and/or references you consult.

Exercise 6.1.

- Let X be a copy of the Möbius band. Let $A = \partial X$ be the topological boundary of X. Find a Δ-complex structure on X and thus on A.
- Let X be a copy of the two-torus, minus a small open disk. Let $A = \partial X$ be the topological boundary of X. Find a Δ-complex structure on X and thus on A.

In each of the two cases above: compute the homology groups of A and of X, the maps on homology induced by inclusion, and the relative homology groups of the pair (X, A).

Exercise 6.2.

- Show if (X, A) is a good pair then so is $(X/A, A/A)$. [Harder] What about the converse?
- Suppose H is the Hawaiian earring and 0 is the origin. Show $(H, \{0\})$ is not a good pair.

Exercise 6.3. Prove the inclusions $(B^n, S^{n-1}) \subset (B^n, B^n - \{0\}) \subset (\mathbb{R}^n, \mathbb{R}^n - \{0\})$ induce isomorphisms of relative homologies. (We used this in our proof of invariance of domain.)

Exercise 6.4. [Medium. See Hatcher page 133, problem 27, for definitions.] Show the inclusion $i\colon (B^n, S^{n-1}) \to (B^n, B^n - \{0\})$ is not a homotopy equivalence of pairs. (This rules out one possible approach to Exercise 6.3.)

Exercise 6.5. Suppose $f\colon B^n \to B^n$ is a fixed-point-free map. For any $x \in B^n$, let L_x be the straight line through x and $f(x)$, oriented from x towards $f(x)$. For points $y, z \in L_x$ we write $y < z$ if y is before z according to the orientation on L_x. Let $g(x)$ and $h(x)$ be the two points of $L_x \cap S^{n-1}$, where $g(x) \leq x < f(x) \leq h(x)$. Prove $g\colon B^n \to S^{n-1}$ is well-defined, continuous, and a retraction. (This is a step of the proof of the Brouwer fixed point theorem.)

Exercise 6.6. [Hard. Hatcher page 133, problems 20 and 21.] Define CX, the *cone* of X, to be $X \times I / X \times \{1\}$. Define SX, the *suspension* of X, to be the space obtained by doubling CX across its base. That is, SX is obtained from $X \times [-1, 1]$ by crushing $X \times \{-1\}$ and $X \times \{1\}$ to points.

- Prove SS^n is homeomorphic to S^{n+1}.
- Find chain maps $s_n\colon C_n(X) \to C_{n+1}(SX)$ that induce isomorphisms $s_{n*}\colon H_n(X) \to H_{n+1}(SX)$.