Please let me know if any of the problems are unclear or have typos. Please turn in a *single* worked exercise — write your name, the date, and the problem you are solving at the top of the page. If you collaborate with other students, please also include their names.

Exercise 3.1. Compute the simplicial homology groups of the two-sphere, directly from the definitions, using the Δ–complex structure coming from the boundary of a tetrahedron. Provide clearly labelled figures.

Exercise 3.2. Suppose that X is a path-connected space, equipped with a Δ–complex structure. Show, directly from the definitions, that $H_0^\Delta(X) \cong \mathbb{Z}$. (You may assume without proof that the one-skeleton is connected.)

Exercise 3.3. Suppose that X is a finite, path-connected, one-dimensional Δ–complex: that is, a finite connected graph. Suppose that X has E edges and V vertices. Compute the simplicial homology groups of X.

Exercise 3.4. [Do not turn in.] Compute the reduced singular homology groups of a point, directly from the definitions.

Exercise 3.5. [Challenge.] Compute the singular homology groups of the circle S^1, directly from the definitions.

Exercise 3.6. Compute the singular homology groups of the plane $\mathbb{R}^2$, minus n points. Reference any theorems from Hatcher that you use.

Exercise 3.7. Suppose $f_\#\colon \mathcal{C}_* \to \mathcal{D}_*$ is a *chain map* of chain complexes: a sequence of group homomorphisms $\{f_n\}$ so that $\partial_n^D \circ f_n = f_{n-1} \circ \partial_n^C$. The short-hand for this is $\partial f_\# = f_\# \partial$. Show $f_\#$ induces well-defined homomorphisms $f_*\colon H_*(\mathcal{C}) \to H_*(\mathcal{D})$ on homology.

Exercise 3.8. [Do not turn in.]

- Suppose that X is a Δ–complex. Let $i_\#\colon \mathcal{C}_*^\Delta(X) \to \mathcal{C}_*^s(X)$ be the injective homomorphisms sending a simplex of the Δ–complex to the corresponding singular simplex. Show that $i_\#$ is a chain map. (Later in the course we will prove $i_\#$ induces an isomorphism from simplicial homology to singular.)

- Suppose that $f\colon X \to Y$ is a map of topological spaces. For any singular n–simplex $\sigma\colon \Delta^n \to X$ define $f_n(\sigma) = f \circ \sigma$. Extend linearly to get a homomorphism $f_n\colon C_n^s(X) \to C_n^s(Y)$. Show that this collection $f_\# = \{f_n\}$ is a chain map.

Exercise 3.9. [Hatcher problem 12, page 132.] Two chain maps $f_\#$ and $g_\#$ from $\mathcal{C}_*$ to $\mathcal{D}_*$ are *chain homotopic* if there is a sequence of homomorphisms $P_n\colon C_n \to D_{n+1}$ so that

$$\partial_{n+1}^D \circ P_n + P_{n-1} \circ \partial_n^C = g_n - f_n$$

for all n. The short-hand for this is $\partial P + P\partial = g_\# - f_\#$. We call P a *chain homotopy* and write $f_\# \sim g_\#$. Show chain homotopy of chain maps is an equivalence relation.

Exercise 3.10. We say two chain complexes $\mathcal{C}_*$ and $\mathcal{D}_*$ are *chain homotopy equivalent* if there are chain maps $f_\#\colon \mathcal{C}_* \to \mathcal{D}_*$ and $g_\#\colon \mathcal{D}_* \to \mathcal{C}_*$ so that $g_\# \circ f_\# \sim \mathrm{Id}_C$ and $f_\# \circ g_\# \sim \mathrm{Id}_D$.

Let $\mathcal{C}_*$ be the chain complex with $C_1 = C_0 = \mathbb{Z}$, all other chain groups trivial, and with $\partial_1(m) = 2m$. Let $\mathcal{D}_*$ be the chain complex with $D_1 = D_0 = \mathbb{Z}^2$, all other chain groups trivial, and with $\partial_1(x, y) = (x - y, x + y)$. Prove $\mathcal{C}_*$ and $\mathcal{D}_*$ are chain homotopy equivalent.