Please let me know if any of the problems are unclear or have typos.

Exercise 8.1. [Do not turn in.] Suppose that G is a connected graph. Show that G is path-connected. Deduce that for any vertices $x, y \in G$ there is a finite embedded edge-path connecting x to y.

Exercise 8.2. A graph T is a *tree* if it is contractible. A graph T is *tree-like* if it is connected and contains no embedded circle.

- Suppose that G is a graph and $L \subset G$ is an embedded circle. Show that G retracts to L.
- Show that trees are tree-like.

Exercise 8.3. Suppose that T is tree-like.

- If T is also a finite graph then either T is a single point or T has a *leaf*: a vertex meeting a single end of a single edge.
- Fix a pair of distinct points $x, y \in T$. Show that there is a unique edge-path (necessarily finite) connecting x to y. This edge-path is denoted by $[x, y] \subset T$.
- Show that tree-like graphs are trees.

Exercise 8.4. [Medium.] Suppose that G is a connected graph. Show that G contains a *maximal tree*: a subgraph $T \subset G$ so that T is a tree and T contains all vertices of G.

Exercise 8.5. Suppose that G is a connected graph and $T \subset G$ is a maximal tree where $G - T$ consists of a single edge e. Show that $\pi_1(G) \cong \mathbb{Z}$.

Exercise 8.6. Suppose that G is a connected graph. Show that $\pi_1(G)$ is a free group. Now compute the fundamental group of each of the graphs below.