

Please let me know if any of the problems are unclear or have typos.

**Exercise 2.1.** Show that the relation  $f \simeq g$  of being homotopic is an equivalence relation on maps.

**Exercise 2.2.** Show that the relation  $X \simeq Y$  of being homotopy equivalent is an equivalence relation on topological spaces.

**Exercise 2.3.** Show that  $\mathbb{R}^n - \{0\} \cong S^{n-1} \times \mathbb{R} \simeq S^{n-1}$ . That is, the first pair of spaces are homeomorphic while the second pair are homotopy equivalent. Use this to prove that  $\mathbb{R}^n - \{0\} \simeq S^{n-1}$ .

**Exercise 2.4.** Fix  $m, n \in \mathbb{N}$  so that  $0 < n < m$ . We embed  $\mathbb{R}^n$  into  $\mathbb{R}^m$  by taking  $(x_1, \dots, x_n) \in \mathbb{R}^n$  to  $(x_1, \dots, x_n, 0, \dots, 0) \in \mathbb{R}^m$ . Show that  $\mathbb{R}^m - \mathbb{R}^n \simeq S^{m-n-1}$ .

**Exercise 2.5.** [Medium] Consider the capital letters of the alphabet A, B, C, ... in a sans serif font. Each of these gives a graph in the plane. Sort these into homotopy equivalence classes. Clearly state any unproven assumptions that you rely on.

**Exercise 2.6.** [Medium] Show that the eyeglasses graph  $E$  and the theta graph  $T$  (both shown in Figure 1) are homotopy equivalent.

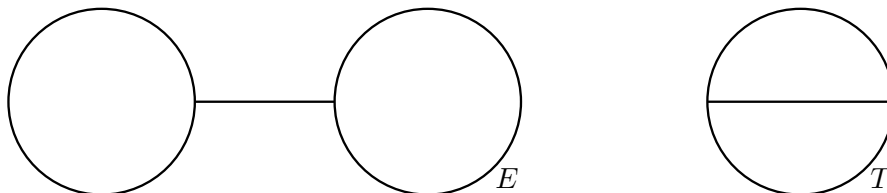


Figure 1: Left: the eyeglasses graph. Right: the theta graph.