Please let me know if any of the problems are unclear or have typos.

Exercise 9.1. A metric space (X, d_X) is *homogeneous* if for all $p, q \in X$ there is an isometry $T \in \mathrm{Isom}(X)$ so that $T(p) = q$. Show that the three geometries S^2, $\mathbb{E}^2$, and $\mathbb{H}^2$ are all homogeneous. In each case, discuss the uniqueness of the isometry T.

Exercise 9.2. Show that for any pair $u, v \in \mathbb{L}^3$ of light-like vectors there is a matrix $A \in O^+(1,2)$ with $Au = v$. Prove or disprove: the matrix A is unique.

Exercise 9.3. Suppose that $\Delta, \Delta' \subset \mathbb{H}^2$ are *ideal triangles*: that is, the three sides of Δ are infinite, pairwise asymptotic lines. Show that there is an isometry $T \in \mathrm{Isom}(\mathbb{H}^2)$ so that $T(\Delta) = \Delta'$. Prove or disprove: the isometry T is unique.

Exercise 9.4. [Medium.] Suppose that $\Delta, \Delta' \subset \mathbb{H}^2$ are triangles where all side lengths are finite. Suppose also that Δ and Δ' have interior angles $\alpha = \alpha'$, $\beta = \beta'$, and $\gamma = \gamma'$. Finally assume the angles α, β, and γ are all distinct. Show that there is an isometry $T \in \mathrm{Isom}(\mathbb{H}^2)$ so that $T(\Delta) = \Delta'$. Prove or disprove: the isometry T is unique.

Exercise 9.5. [Do not hand in.] Exercise 4.4 from Chapter 4.

Exercise 9.6. Suppose that $T(x) = Ax + v$, where $A \in \mathrm{GL}(n, \mathbb{R})$ and $v \in \mathbb{R}^n$. Show that T sends affine subspaces of $\mathbb{A}^n$ to affine subspaces.