Please let me know if any of the problems are unclear or have typos.

Exercise 8.1. Suppose that $B, C \in M_3(\mathbb{R})$ are three-by-three matrices over $\mathbb{R}$. Suppose that there is an open set $U \subset \mathbb{R}^3$ so that for all vectors $u, v \in U$ we have $u^T B v = u^T C v$. Prove that $B = C$.

Exercise 8.2. Set $a_\pm = (1, \pm 1, 0)$ and $b_\pm = (1, 0, \pm 1)$. Thus $M = \mathbb{H}^2 \cap \operatorname{span}(a_+, b_+)$ and $N = \mathbb{H}^2 \cap \operatorname{span}(a_-, b_-)$ are hyperbolic lines. Show that $M \cap N = \emptyset$. Find points $P \in M$ and $Q \in N$ so that the hyperbolic line $L = \overline{PQ}$ is orthogonal to both M and N.

Exercise 8.3. We define the following functions from $\mathbb{R}$ to $M_3(\mathbb{R})$.

$$\operatorname{Rot}_t = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(t) & -\sin(t) \\ 0 & \sin(t) & \cos(t) \end{pmatrix} \quad \operatorname{Tran}_t = \begin{pmatrix} \cosh(t) & \sinh(t) & 0 \\ \sinh(t) & \cosh(t) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\operatorname{Para}_t = \begin{pmatrix} 1 + t^2/2 & -t^2/2 & t \\ t^2/2 & 1 - t^2/2 & t \\ t & -t & 1 \end{pmatrix}$$

Show that $\operatorname{Rot}_0 = \operatorname{Id}$, that $\operatorname{Rot}_{s+t} = \operatorname{Rot}_s \operatorname{Rot}_t$, and that $\operatorname{Rot}_t \in O^+(1, 2)$. Now do the same for Tran and Para.

Exercise 8.4. Set $L(t) = (\cosh(t), \sinh(t), 0)$. Set $A = \operatorname{Para}_1$, using the notation of Exercise 8.3. Let $L'(t) = A(L(t))$. That is, L' is the image of the line L under the hyperbolic isometry A. Show that L and L' are disjoint. Now compute the distance $d_{\mathbb{H}}(L(t), L'(t))$ for all t. How does the distance behave as t tends to negative infinity? To positive infinity?

Exercise 8.5. Suppose that $S \in \mathbb{L}^3$ is a space-like vector of length one. Define $L = S^\perp \cap \mathbb{H}^2$. Define $\operatorname{Refl}_L(P) = P - 2(P \circ S)S$.

- Show that $\operatorname{Refl}_L(\operatorname{Refl}_L(P)) = P$ for all $P \in \mathbb{H}^2$.
- Show that $\operatorname{Refl}_L(Q) = Q$ for all $Q \in L$.
- Suppose that $P \in \mathbb{H}^2$ does not lie on L. Show that the line through P and $Q = \operatorname{Refl}_L(P)$ is orthogonal to L.
- Suppose that L and L' are hyperbolic lines that intersect in $\mathbb{H}^2$. Describe the composition of Refl_L with $\operatorname{Refl}_{L'}$.