Please let me know if any of the problems are unclear or have typos.

Exercise 1.1. (See also problem A.2 in [RS].) Fix $n \in \mathbb{Z}$ a positive integer. Let $X = \{0,1\}^n$ be the set of binary strings of length n. For strings $x, y \in X$ define the *Hamming distance* $d(x, y)$ to be the number of positions i so that $x_i \neq y_i$. For example, when $n = 4$ the Hamming distance between 0101 and 0011 is two. Prove (X, d) is a metric space. Also, compute the *diameter* of X: the largest possible distance between a pair of points of X.

Exercise 1.2. (Problem A.1 of [RS].) Suppose (X, d_X) and (Y, d_Y) are metric spaces. We call a function $f\colon X \to Y$ an *isometric embedding* if for all $x, y \in X$ we have $d_Y(f(x), f(y)) = d_X(x, y)$. Justify the terminology by proving f is injective.

Show, by means of an example, that f need not be surjective even when $X = Y$.

Exercise 1.3. Suppose (X, d_X) and (Y, d_Y) are metric spaces. An *isometry* $f\colon X \to Y$ is an isometric embedding that is, additionally, surjective. Let $\mathrm{Isom}(X)$ be the set of isometries from X to itself. Show $\mathrm{Isom}(X)$ is a group, if we take the group operation to be function composition.

Exercise 1.4. Let $d\colon \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ be the usual metric on the real line: namely $d(x, y) = |x - y|$. Compute the group $\mathrm{Isom}(\mathbb{R})$, where $\mathrm{Isom}(X)$ is defined as in Exercise 1.3.

Exercise 1.5. Recall the statement of the Cauchy-Schwarz inequality: For any $u, v \in \mathbb{R}^n$ we have $|u \cdot v| \leq |u||v|$. Furthermore, equality holds if and only if u and v are linearly dependent.

Here is a sketch of Schwarz's proof of the Cauchy-Schwarz inequality. Consider the function $f\colon \mathbb{R} \to \mathbb{R}$ defined by $f(t) = |u + tv|^2$. Thus f is non-negative, and is a quadratic polynomial. Thus the discriminant of f is non-positive. Finally, f vanishes if and only if u is a multiple of v.

Fill in the missing details.