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Brilliant geometry


What is 4-dimensional space?

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We describe a point in four-dimensional space using four numbers, say
 ( $w, x, y, z$ ).

Example: how to make a hypercube

## Example: how to make a hypercube



$\downarrow$
$\downarrow$


## Example: how to make a hypercube



Example: how to make a hypercube


How can we see 4-dimensional things?


## Stereographic projection



First radially project the cube to the sphere...


First radially project the cube to the sphere...


Then stereographically project to the plane


Then stereographically project to the plane


Do the same thing one dimension up to see a hypercube


Do the same thing one dimension up to see a hypercube


More amazing properties of stereographic projection


Regular Polytopes in 2-dimensions: Regular polygons


Regular Polytopes in 3-dimensions: Regular polyhedra


Regular Polytopes in 4-dimensions: Regular polychora


5-cell


24-cell


8-cell


16 -cell


120-cell

$600-c e l l$

Regular Polytopes in 4-dimensions: Regular polychora


5-cell


24-cell


8-cell


16-cell


120-cell

$600-c e l l$

## Thanks!


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