Makeup Final Exam for Math 181, Spring 2003. Show all work in test booklet. There are ten problems (two on back) worth ten points each.

- (1) A rock is thrown straight up from the top of a 64-foot tower with an initial upwards velocity of 32 ft/sec. (Recall g = 32 ft/sec².)
 - (a) When does the rock hit the ground?
 - (b) How fast is it going when it hits the ground?
- (2) Find a function f(x) that satisfies $f'(x) = xe^{-x}$ and f(0) = 0.
- (3) Compute the following integrals.
 - (a) $\int_0^{\pi/4} \tan\theta \sec^2\theta \ d\theta.$ (b) $\int x(x+1)^{100} dx.$ (c) $\int \frac{t^2+1}{t^2-1} dt.$
- (4) Estimate $\int_{-1}^{1} (1-t^2) dt$ using the midpoint rule and subdividing the interval [-1, 1] into three equal subintervals. Is this estimate an over or under-approximation?
- (5) Let $p(t) = e^{-t}$, for t greater than or equal to zero.
 - (a) Check that p is a probability density.
 - (b) What is the median?
 - (c) What is the mean?
- (6) The region bounded by the curves $y = x^2 + 1$, x = 0, and y = 2 is rotated around the y-axis.
 - (a) Draw a picture of the resulting solid of revolution.
 - (b) Express the volume of a thin slice, parallel to the xz-plane, in terms of y and Δy .
 - (c) Use this to express the volume of the solid as an integral.
- (7) Test the following series for convergence.

(a)
$$\sum_{n=0}^{\infty} \frac{1}{2+3^n}$$

(b)
$$\sum_{n=0}^{\infty} \frac{n+4}{n}$$

(c)
$$\sum_{n=1}^{\infty} \frac{1}{2}$$

- (c) $\sum_{n=1}^{\infty} \overline{n^n}$
- (8) Find the radius of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{x^n}{2^n \cdot n^2}$$

- (9) Suppose f(x) = -1 on $(-\pi, 0]$, f(x) = 1 on $(0, \pi]$, and also f is periodic with period 2π . Sketch f(x). Find the first three nonzero terms in the Fourier series for f.
- (10) Let $f(x) = e^x$ on the interval [0, 1]. Find a Taylor polynomial which approximates f(x) with error less than 10^{-10} . Explain your answer.
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