Makeup Final Exam for Math 181, Spring 2003. Show all work in test booklet. There are ten problems (two on back) worth ten points each.
(1) A rock is thrown straight up from the top of a 64 -foot tower with an initial upwards velocity of $32 \mathrm{ft} / \mathrm{sec}$. (Recall $g=32 \mathrm{ft} / \mathrm{sec}^{2}$.)
(a) When does the rock hit the ground?
(b) How fast is it going when it hits the ground?
(2) Find a function $f(x)$ that satisfies $f^{\prime}(x)=x e^{-x}$ and $f(0)=0$.
(3) Compute the following integrals.
(a) $\int_{0}^{\pi / 4} \tan \theta \sec ^{2} \theta d \theta$.
(b) $\int x(x+1)^{100} d x$.
(c) $\int \frac{t^{2}+1}{t^{2}-1} d t$.
(4) Estimate $\int_{-1}^{1}\left(1-t^{2}\right) d t$ using the midpoint rule and subdividing the interval $[-1,1]$ into three equal subintervals. Is this estimate an over or under-approximation?
(5) Let $p(t)=e^{-t}$, for $t$ greater than or equal to zero.
(a) Check that $p$ is a probability density.
(b) What is the median?
(c) What is the mean?
(6) The region bounded by the curves $y=x^{2}+1, x=0$, and $y=2$ is rotated around the $y$-axis.
(a) Draw a picture of the resulting solid of revolution.
(b) Express the volume of a thin slice, parallel to the $x z$-plane, in terms of $y$ and $\Delta y$.
(c) Use this to express the volume of the solid as an integral.
(7) Test the following series for convergence.
(a) $\sum_{n=0}^{\infty} \frac{1}{2+3^{n}}$
(b) $\sum_{n=0}^{\infty} \frac{n+4}{n}$
(c) $\sum_{n=1}^{\infty=0} \frac{1}{n^{n}}$
(8) Find the radius of convergence of the power series

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\sum_{n=1}^{\infty} \frac{x^{n}}{2^{n} \cdot n^{2}}
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(9) Suppose $f(x)=-1$ on $(-\pi, 0], f(x)=1$ on $(0, \pi]$, and also $f$ is periodic with period $2 \pi$. Sketch $f(x)$. Find the first three nonzero terms in the Fourier series for $f$.
(10) Let $f(x)=e^{x}$ on the interval $[0,1]$. Find a Taylor polynomial which approximates $f(x)$ with error less than $10^{-10}$. Explain your answer.

