MA3A6 WEEK 8 ASSIGNMENT : DUE MONDAY 4PM WEEK 8

BILL HART

1. Find out the algorithm used by Pari to compute rings of integers of number fields. You may like to look at the Pari documentation, the source code for Pari (both available from the Pari website - linked to from the class website) or post a question on the Pari support list.

If you decide to look at the source code, you need to look in the directory src/basemath/ I think the files base1, base2, ... are the ones that have the number field functions in them. There may not be much in the way of helpful comments there, but try googling some of the terms used and see if that gives any leads.

The ring of integers is also called the *maximal order* of the number field. It might end up being easier to look for algorithms for computing maximal orders, then try and figure out which one Pari uses!

2. Write a *short* Pari program to compute rings of integers of the fields generated by one of the roots of each of the polynomials $f(x) = x^3 - 3x + i$ for all $1 \leq i \leq 100$.

Write down the number of characters that your program takes to type into Pari (not counting spaces, tabs and returns). The names of the authors of the shortest program will be put on the class website (if the authors approve). Any particularly elegant programs will also be posted.

Note that to loop over all those number fields, you will probably need to use a *for loop*. For example, to print the first 10 primes, I would write:

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for(i=1,10,print("Prime number ", i, " is: ", prime(i)))
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3. Determine which algebraic numbers of the form $\frac{1}{3}(\lambda_1 + \lambda_2\omega + \lambda_3\omega^2)$ can be algebraic integers for $\lambda_i \in \mathbb{Z}$, $0 \leq \lambda_i \leq 2$ and $\omega^3 = 7$.

4. The ring of integers of $\mathbb{Q}(\zeta_5)$ for ζ_5 a primitive 5-th root of unity, is $\mathbb{Z}[\zeta_5]$. Compute the discriminant of $\mathbb{Q}(\zeta_5)$. Check your answer with Pari.

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