MA3A6 WEEK 5 ASSIGNMENT : DUE MONDAY 4PM WEEK 5

BILL HART

1. Determine all the field conjugates of $\beta = 1 + \sqrt{\frac{1+\sqrt{5}}{2}}$ for the number field $K = \mathbb{Q}(\alpha)$ where α is a root of $f(x) = x^4 - x^2 - 1$.

2. Let K be a number field and let $\sigma_1, \sigma_2, \ldots, \sigma_n$ be the monomorphisms from K into $\mathbb{C}$. The absolute norm of an element $\beta \in K$ is defined to be $\mathcal{N}(\beta) = \prod_i \beta_i$ where $\beta_i = \sigma_i(\beta)$.

Let $K = \mathbb{Q}(\alpha)$ where α is a root of $x^4 - x^2 - 1$. Determine $\mathcal{N}\left(\frac{3+\sqrt{5}}{2}\right)$ for the number field K. (Note the norm depends on the number field.)

3. Prove that all quadratic number fields are Galois.

4. A number field is said to be totally real if each of the conjugate roots of the defining polynomial is real.

Consider the cyclotomic number field $\mathbb{Q}(\zeta_p)$ for a prime p, where $\zeta_p^p = 1$ and $\zeta_p \neq 1$.

Let $L = \mathbb{Q}(\beta)$ be the subfield of K generated by the element $\beta = \zeta_p + \zeta_p^{-1}$. Show that L is totally real.

Is L Galois?

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