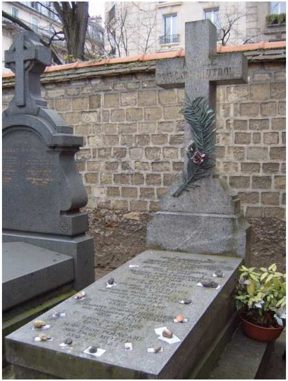
The Poincaré Conjecture

LX 2009

Nicholas Jackson

Easter 2009

"I see dead people"



Cimetière du Montparnasse, Paris

Jules Henri Poincaré (1854–1912)

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If it looks like a sphere, it is.

Definition (n–manifold)

An n–dimensional manifold (or n–manifold) is an object which locally looks like ordinary, flat, Euclidean n–dimensional space ($\mathbb{R}^n$) or half-space ($\mathbb{R}^{n-1} \times \mathbb{R}^+$).

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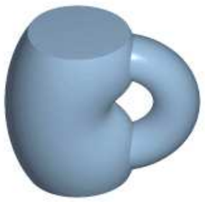

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A topologist is a person who does not know the difference between a doughnut and a coffee cup. – John L Kelley

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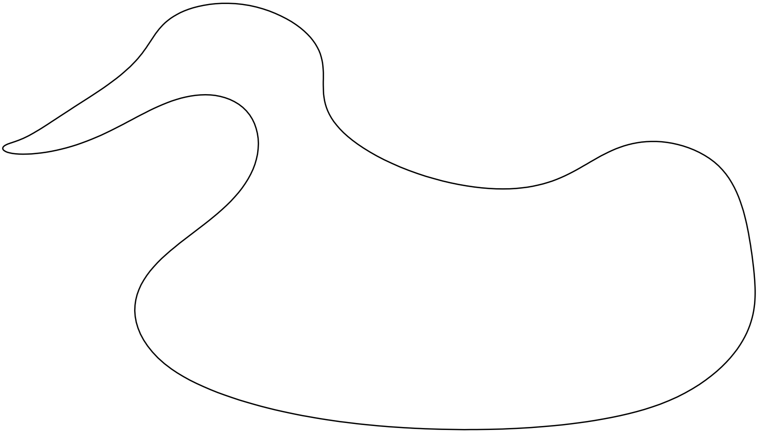

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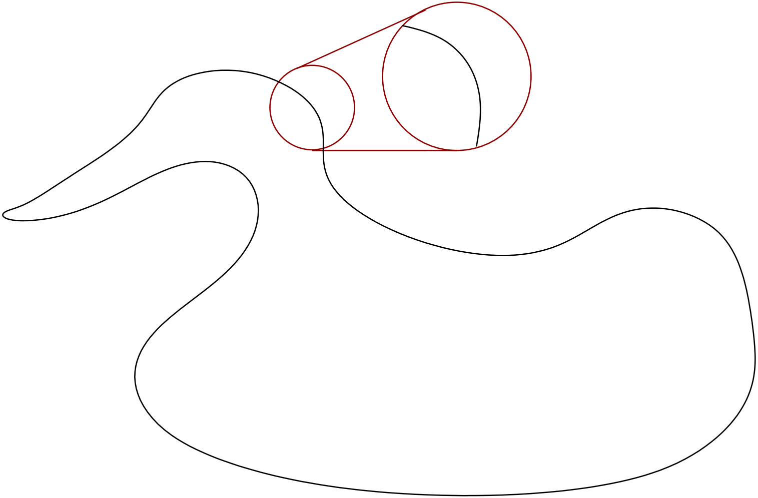

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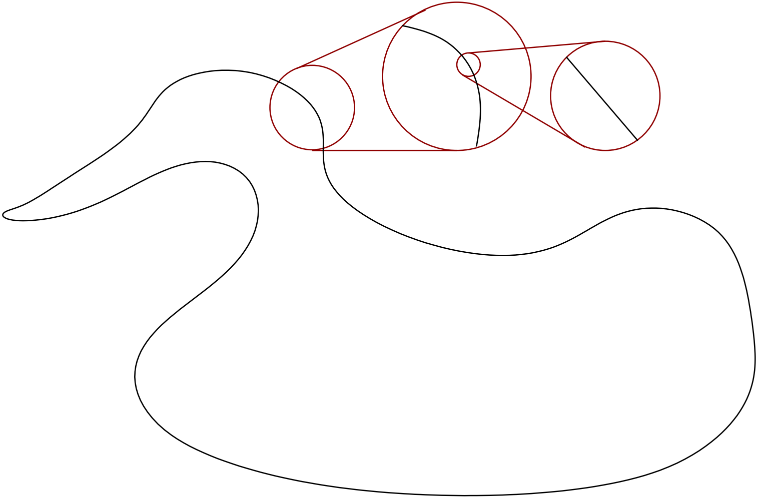

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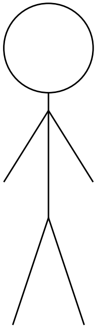

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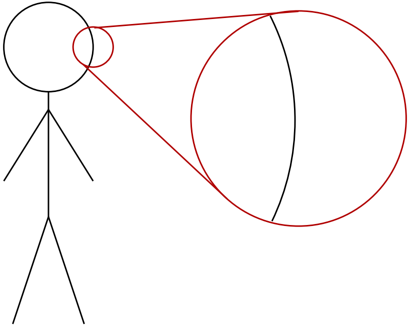

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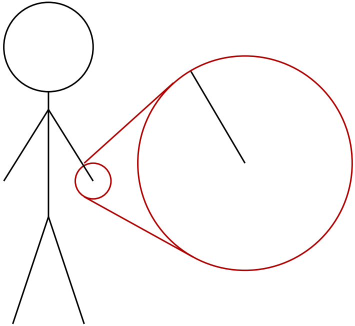

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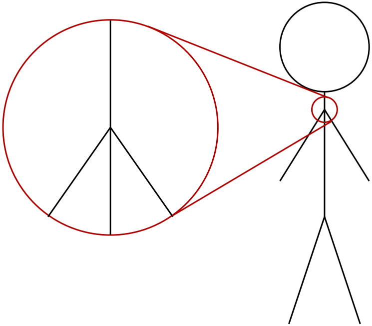

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- An ant walking around on the surface of a football has two degrees of freedom.
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- Flying cars are a very bad idea.

There are two different sorts of 1–manifolds:

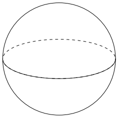
All 1–manifolds are composed of disjoint copies of these.

Corollary

All 1–manifolds have a non-negative, even number of boundary components (ends).

Start with the closed (boundaryless) ones.

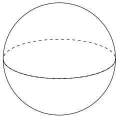
The 2–sphere,



S^2

Start with the closed (boundaryless) ones.

The 2–sphere, the torus,



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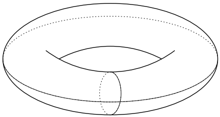

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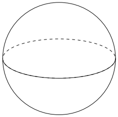

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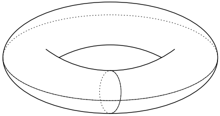

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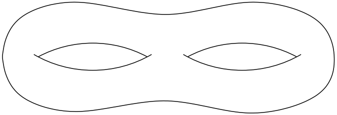

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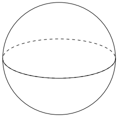

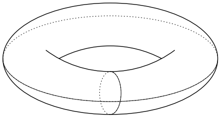

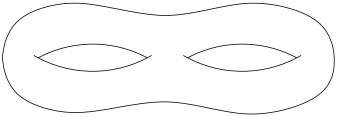

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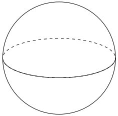

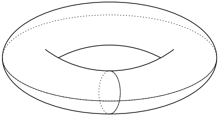

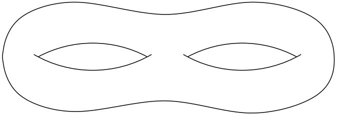

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S_0

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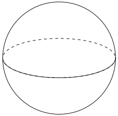	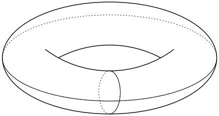	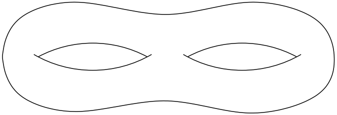	...
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S_0	S_1		

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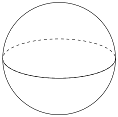
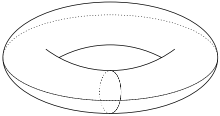
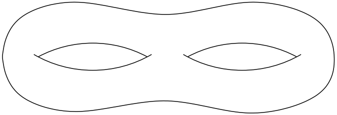
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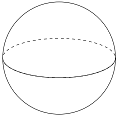	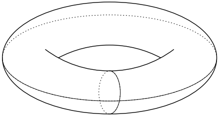	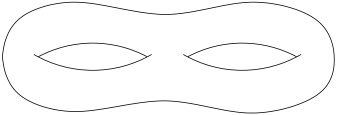	···
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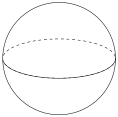

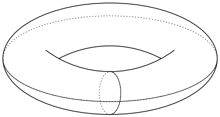

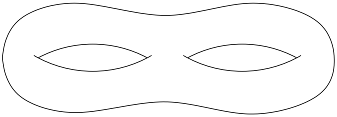

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S_0 $\quad$ S_1 $\quad$ S_2 $\quad$ …

S_g is the surface of genus g (number of toroidal holes).

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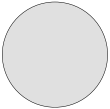

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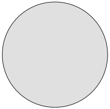

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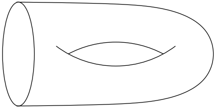

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Classification of 2–manifolds (surfaces)

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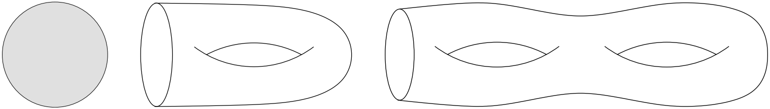

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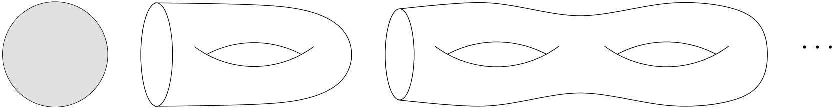

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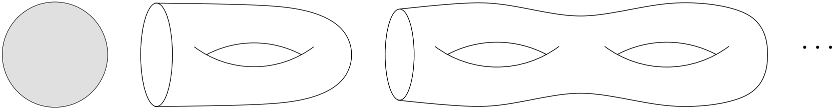

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$S_{0,1}$

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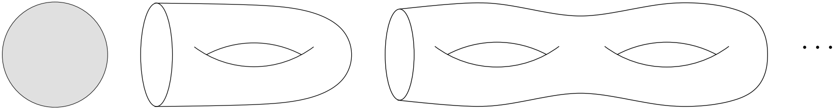

D^2 $T^2 \# D^2$ $T^2 \# T^2 \# D^2$ ...

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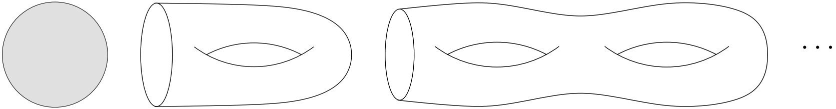

D^2	$T^2 \# D^2$	$T^2 \# T^2 \# D^2$	$\cdots$
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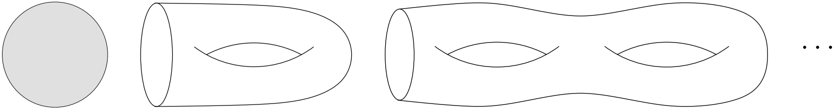

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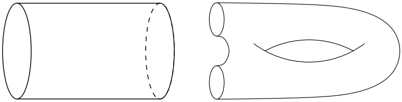

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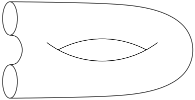

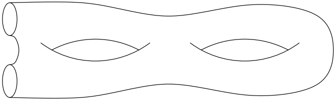

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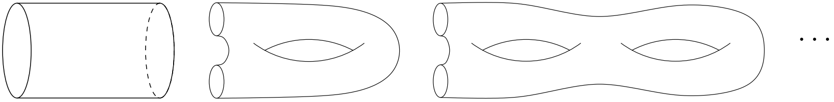

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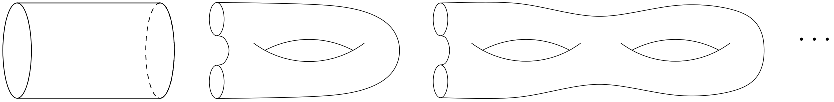

$A^2 = D^2 \# D^2$ | $T^2 \# D^2 \# D^2$ | $T^2 \# T^2 \# D^2 \# D^2$ | ...

$S_{0,2}$ | $S_{1,2}$ | $S_{2,2}$ | ...

$S_{g,b}$ is the surface of genus g with b boundary components.

This isn't the full story...

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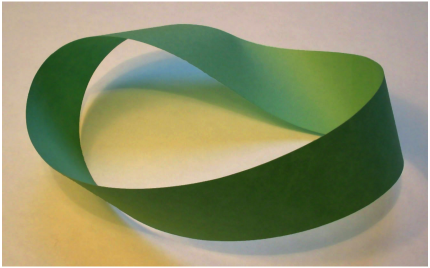

Möbius strip

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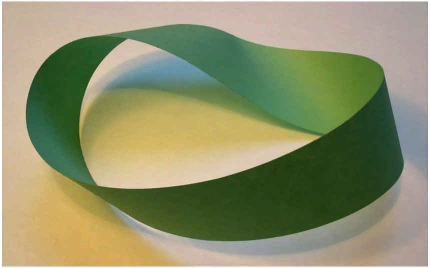

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Klein bottle

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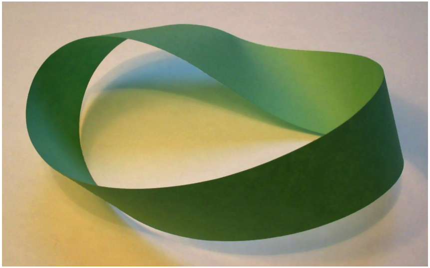
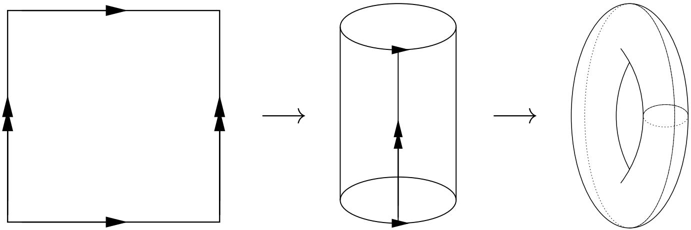


Möbius strip

Klein bottle

The classification theorem can be extended to cover nonorientable surfaces like this.

We can make a torus by identifying (gluing together) the edges of a square in a particular way:



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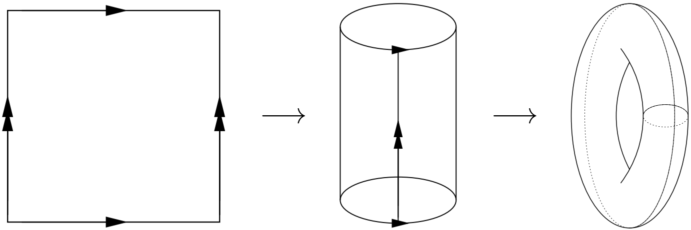

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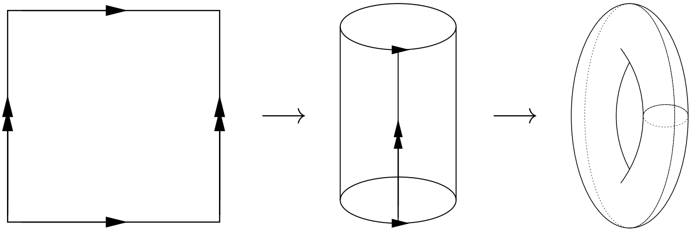
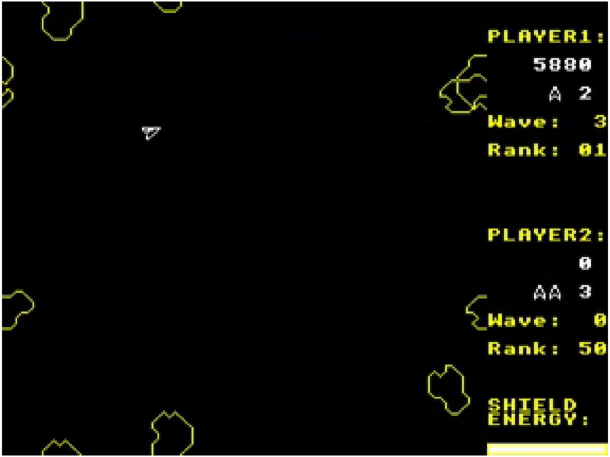


An ant walking around on this square will think it's living on a torus. This may be familiar to those of you who remember the 1980s...

This is just like Asteroids:



We can make any (orientable or nonorientable) surface like this.

Where do we start with 3–manifolds?

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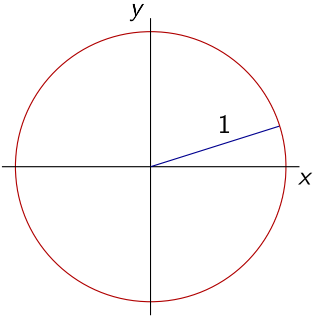
- Therefore the simplest closed 3–manifold should be the 3–sphere S^3.

So what is it?

Extrinsically: $S^1 = \left\{(x,y) : \sqrt{x^2+y^2} = 1\right\} \subset \mathbb{R}^2$

This is the set of points at unit distance from the origin in the plane.



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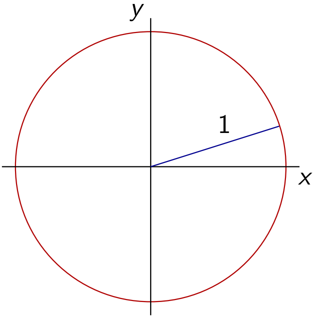

Similarly

$$S^2 = \{(x, y, z) : \sqrt{x^2 + y^2 + z^2} = 1\} \subset \mathbb{R}^3$$

(the set of all points at unit distance from the origin in 3–space).

So, we could define

$$S^3 = \left\{(x, y, z, w) : \sqrt{x^2 + y^2 + z^2 + w^2} = 1\right\} \subset \mathbb{R}^4$$

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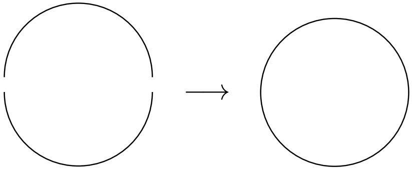

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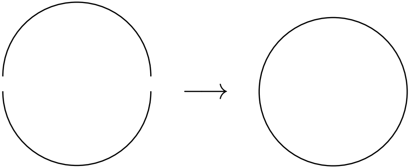
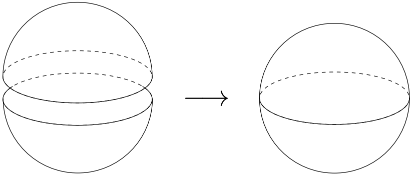
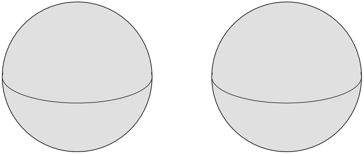


And we can make S^2 by gluing together two discs along their edges:



So, by analogy, we should be able to construct S^3 by gluing together two solid 3–balls along their bounding surfaces.



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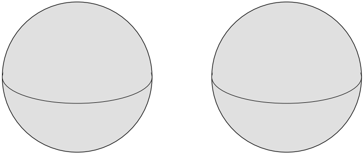
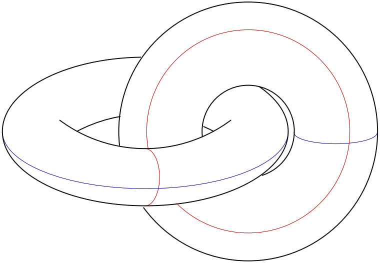
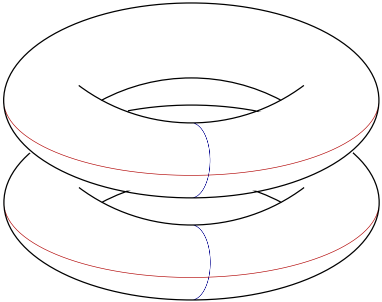


Not very easy to visualise extrinsically, but imagine a 3–dimensional game of Asteroids.

Another way is to glue two solid tori together along their boundaries, so that the meridional circle of one is lined up with the longitudinal circle of the other, and vice versa.



If, on the other hand, we glue the meridional circles to each other, and the longitudinal circles to each other, we get a different 3–manifold ($S^2 \times S^1$) instead:



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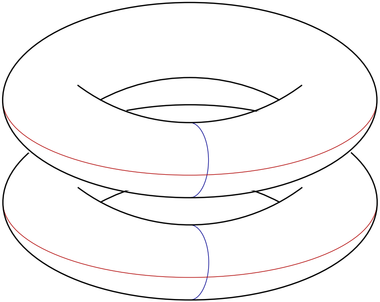
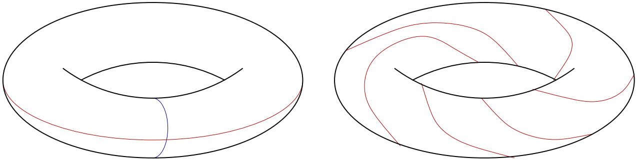


One way to tell that this isn't S^3 is by looking at closed loops in the manifold. S^3 is simply-connected (all closed loops can be shrunk down to a point) but this manifold isn't.

More generally, glue the meridian on one torus to a (p, q)–curve on the other.



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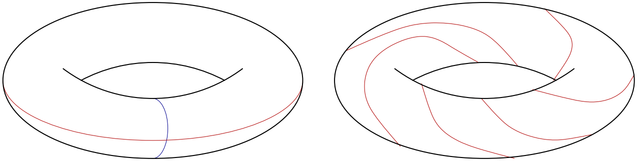

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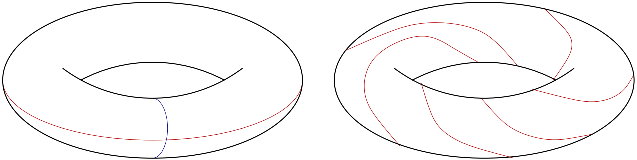

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In particular, $L_{0,1} = S^3$ and $L_{1,0} = S^1 \times S^2$.
In general, $L_{p,q}$ isn't simply-connected: there exist closed loops that can only be shrunk down to a point modulo p.

Another way of looking at this is to say that we can get any lens space $L_{p,q}$ by taking S^3, cutting out a solid torus, and gluing it back in with a twist.

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This procedure is called surgery. If we use a (p, q)–curve it's rational surgery, if we just use a $(p, 1)$–curve it's integer surgery.

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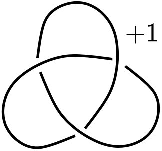
More generally, we can do this with a knotted solid torus (or tori):

Theorem (Lickorish–Wallace Theorem)

Any compact, orientable, closed 3–manifold can be constructed from S^3 by integer surgery on a knot or link.

Integer surgery on a right-handed trefoil with framing $+1$ gives a 3–manifold called the Poincaré homology sphere or Poincaré dodecahedral space.



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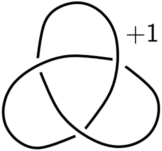
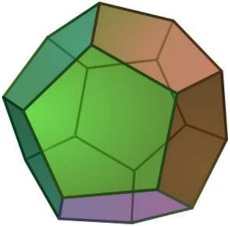


Alternatively, take a solid dodecahedron and identify opposite faces with a 36° twist.



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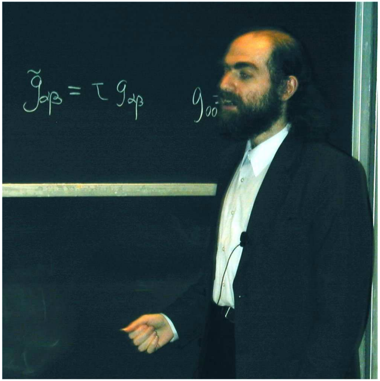
In 1982, William Thurston formulated the Geometrisation Conjecture, which says that all compact 3–manifolds can be decomposed into submanifolds with one of eight geometric structures. The Poincaré Conjecture is a corollary of the Geometrisation Conjecture.

In 2002–2003, the Russian mathematician Grigori Perelman posted three papers on the arXiv containing a proof of the Poincaré Conjecture and the Geometrisation Conjecture, using a technique called Ricci flow devised some years earlier by Richard Hamilton.




PLAYER1:
13160
0
Wave: 4
Rank: 01
GAME OVER
PLAYER2:
0
AA 3
Wave: 0
Rank: 50
SHIELD
ENERGY: