MA 474

THE UNIVERSITY OF WARWICK

FOURTH YEAR EXAMINATION: JUNE 2004

REPRESENTATION THEORY

Time Allowed: 3 hours

Read carefully the instructions on the answer book and make sure that the particulars required are entered.

ANSWER 4 QUESTIONS.

If you have answered more than the required 4 questions in this examination, you will only be given credit for your 4 best answers.

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1. Let A be the subspace of $M_3(\mathbb{C})$ of matrices (a_{ij}) which satisfy $a_{ij} = 0$ for $i < j$ and $a_{11} = a_{33}$.

a) Show that A is an algebra. **[5]**

b) Give the definition of a composition series for a representation. **[5]**

c) Consider the inclusion of A in $M_3(\mathbb{C})$ as a representation and find a composition series. **[5]**

d) Hence find the dimension vector of this representation. **[5]**

e) Find a nilpotent endomorphism of this representation. **[5]**

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2. a) Define the nilpotent radical of an algebra. **[5]**

Let A be the algebra $\mathbb{R}[x]/ < p(x) >$ where $p(x)$ is a polynomial.

b) Find the radical of A for:

(i) $p(x) = (x-2)^2$.

(ii) $p(x) = x^2 + 9$.

[6]

c) Show, that in each case, the radical satisfies the definition you have given in a). **[6]**

d) Give the definition of a local algebra. **[3]**

e) Show that each algebra in b) is a local algebra. **[5]**

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3. a) State the classification of semisimple algebras over $\mathbb{C}$. **[5]**

b) Give the classification of semisimple algebras over $\mathbb{C}$ of dimension at most three. **[6]**

c) Show that there are just five three dimensional algebras over $\mathbb{C}$ (up to isomorphism). You may use any results from the course without proof but these should be clearly stated. **[14]**

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4. a) Give the definition of a projective module. **[6]**

b) Give the definition of a projective cover of a module M. **[7]**

c) Prove that any two projective covers of M are isomorphic. **[12]**

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5. Let A be the quiver algebra of the quiver

$$e \bullet \underset{\beta}{\overset{\alpha}{\rightleftarrows}} \bullet f$$

with the relation $\alpha\beta\alpha = 0$.

a) Find a basis of A where each element is a path. **[3]**

b) Find the simple modules. **[7]**

c) Find the indecomposable projective modules. **[7]**

d) Give the definition of the Cartan matrix of an algebra. **[4]**

e) Find the Cartan matrix of A. **[4]**
