

MA377 EXAMPLE SHEET II

Q. 1. Let A be the complex algebra defined by the multiplication table for the standard basis of the quaternions. Show that A is isomorphic to the complex algebra $M_2(\mathbb{C})$.

Q. 2. Let V be three dimensional Euclidean space equipped with the dot product and vector product. Let A be the real vector space $\mathbb{R} \oplus V$ and define a multiplication by

$$(r, u)(s, v) = (rs - u \cdot v, rv + su + u \times v)$$

Show that A is an algebra. Then show that A is isomorphic to the quaternions.

Q. 3. Show that a field has no proper ideal.

Q. 4. Find all two dimensional algebras over \mathbb{R} (up to isomorphism).

Q. 5. Let A be a finite dimensional algebra. Show that every element of A is either invertible or is a zero divisor.