Workshop: Singularities, coherent structures and their role in intermittent turbulence.

Mathematics Research Centre, University of Warwick.

September 9-17, 2005.

Origin of streaky pattern in near-wall turbulent flow.

Presented by Sergei Chernyshenko,

Aerodynamics and Flight Mechanics Research Group,

School of Engineering Sciences,

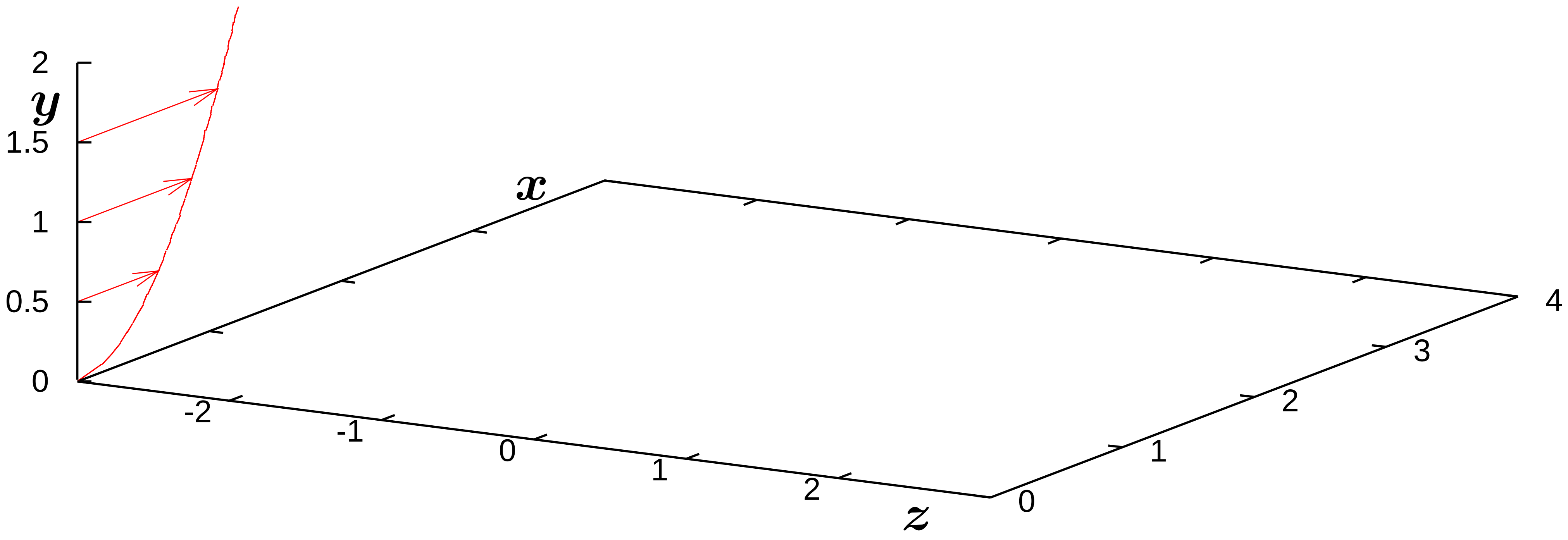
University of Southampton.

Contributions of M.F.Baig and J.Weller are acknowledged.

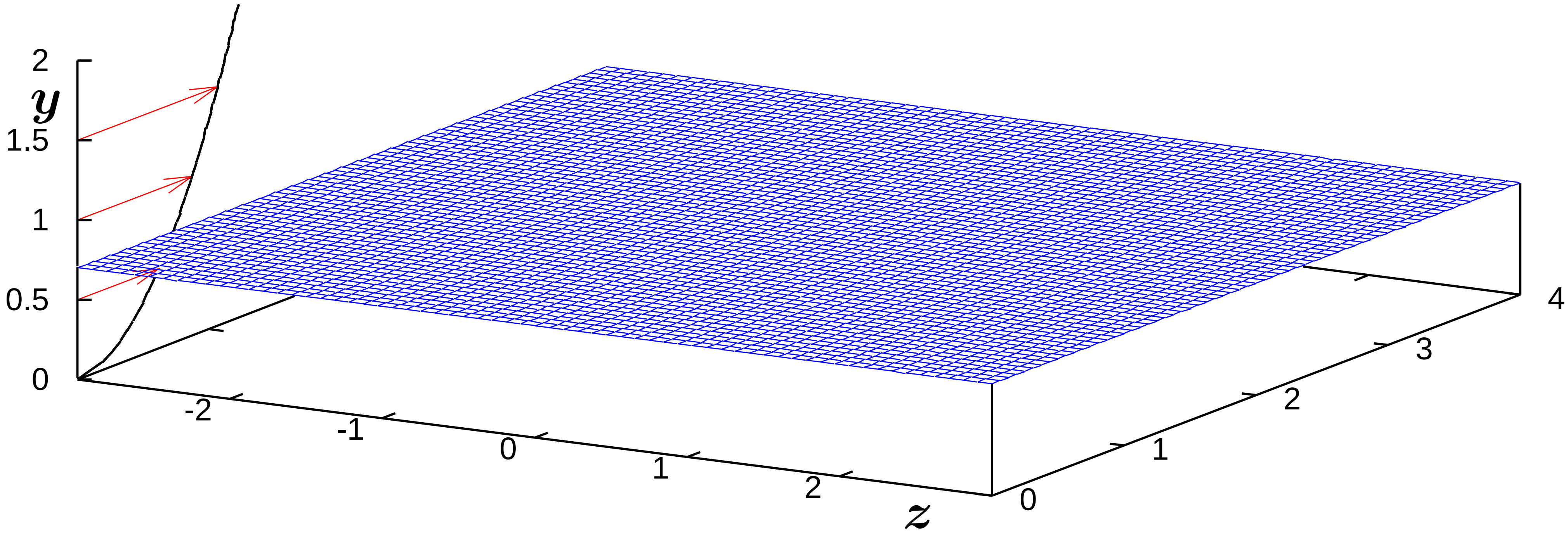
Talk structure:

- Part 1: Conceptual
  - Introduction: streaks
  - Two conceptual frameworks for streaky structure origin: which is true?
  - Numerical experiments
  - Predictive force of the new framework
  - Implications
- Part 2: How to make predictions within the new framework?
  - Generalised optimal perturbations
  - Simplified versions
  - Difficulties
- Part 3: There is work to do!

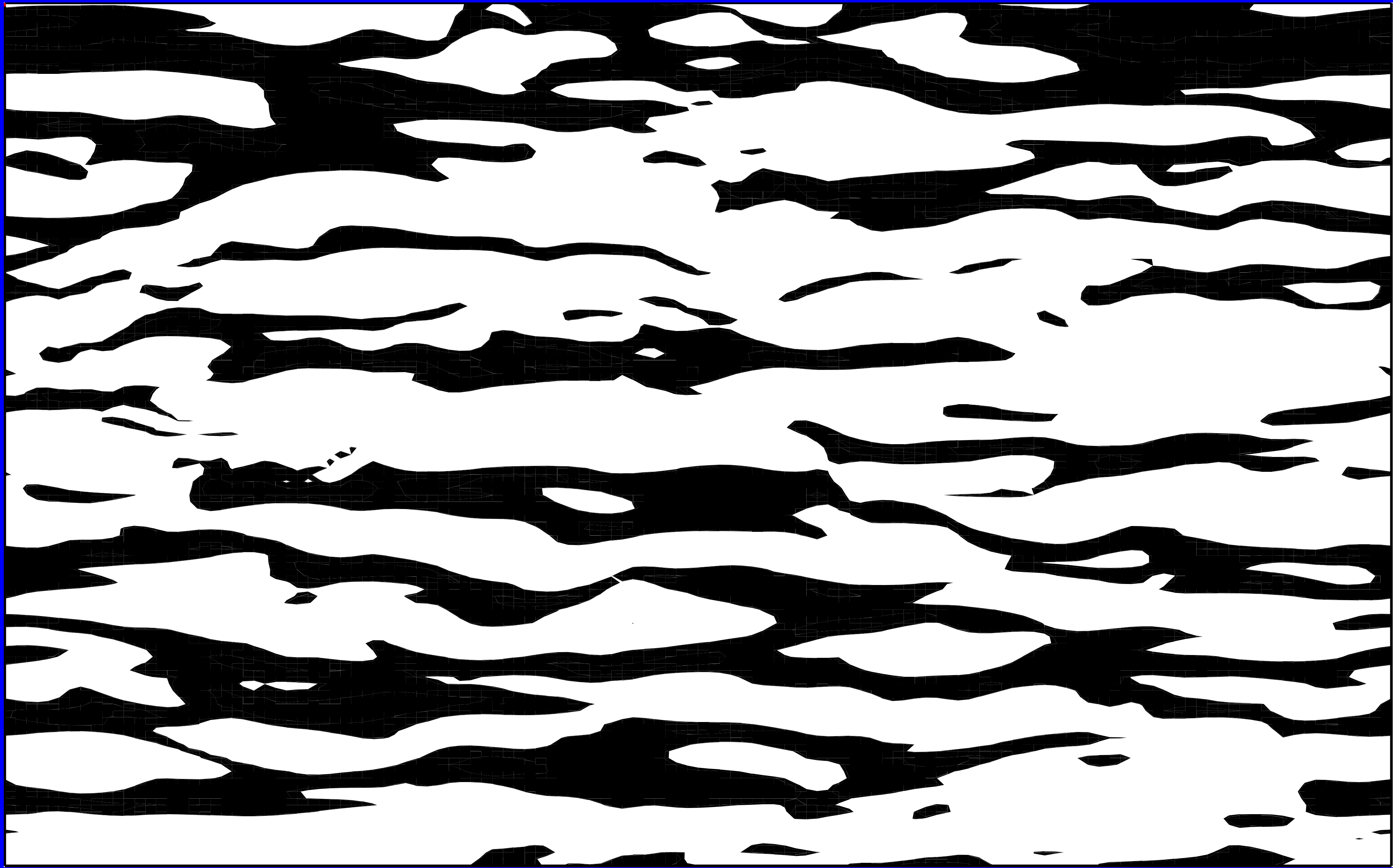
Part 1



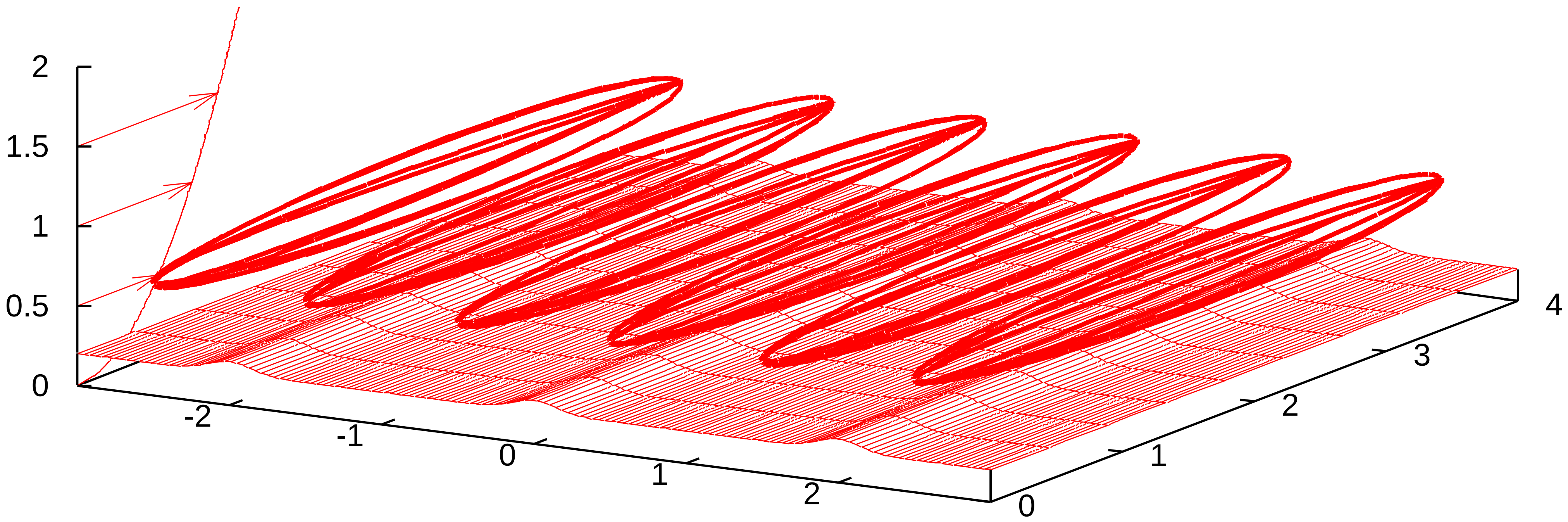
Geometry and the mean profile



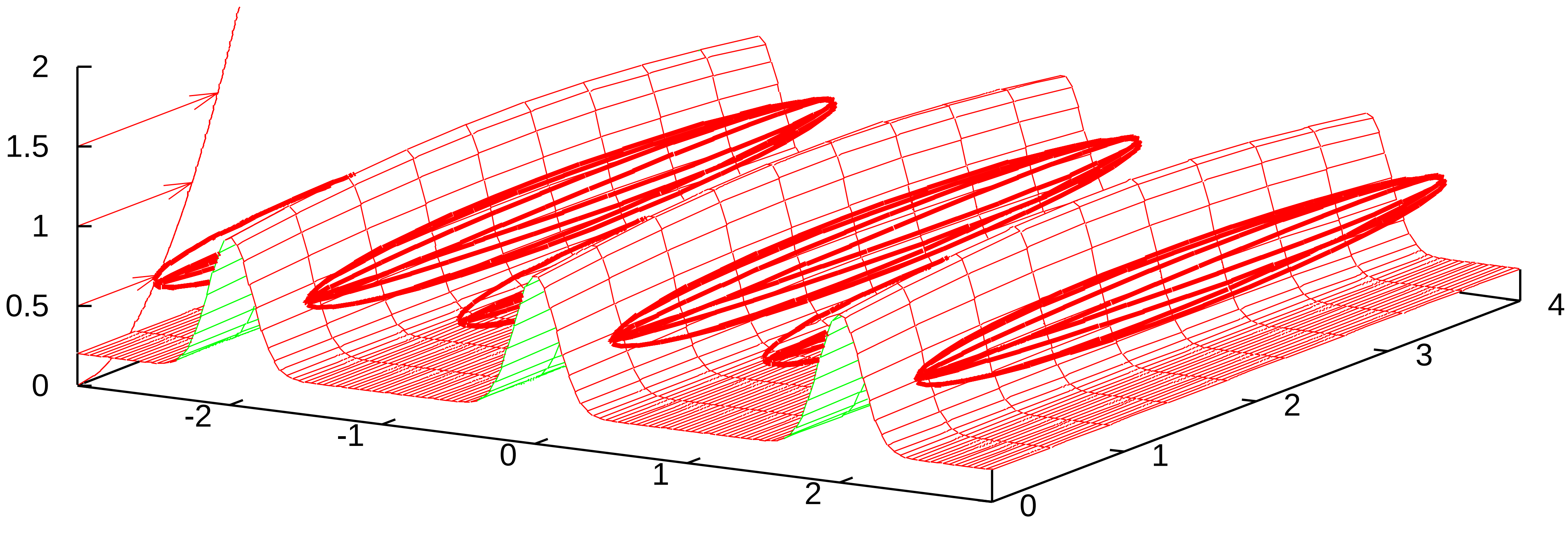
Visualisation plane



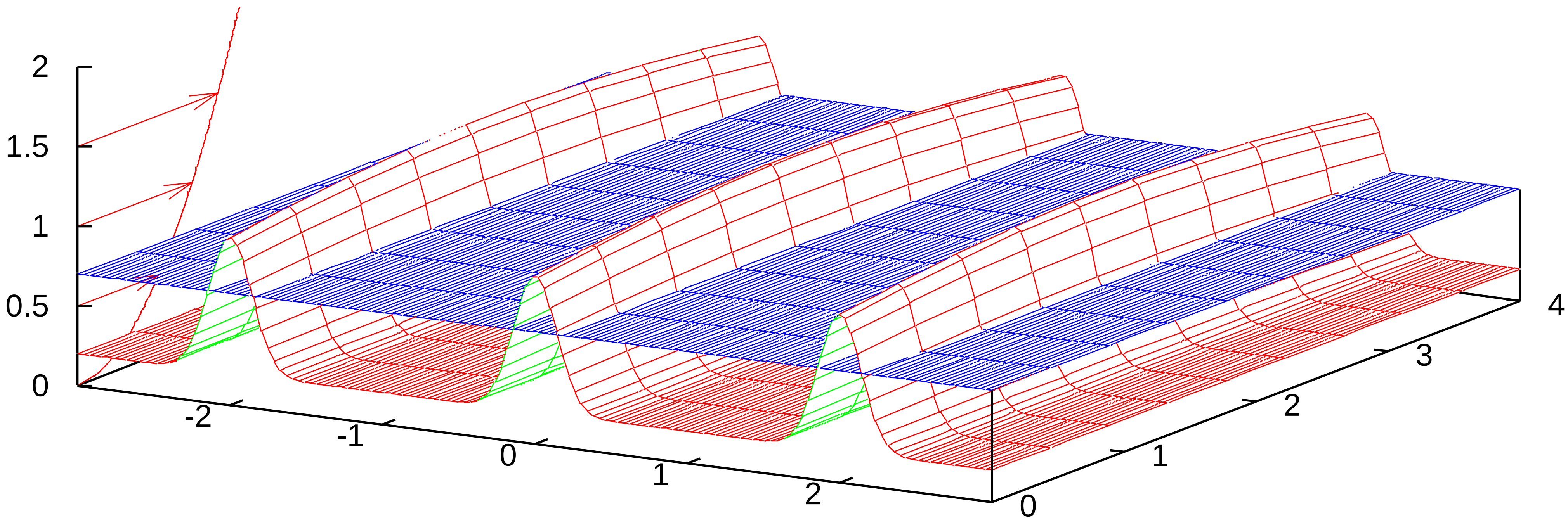
Velocity streaks in a turbulent flow at $y^+ = 5$.



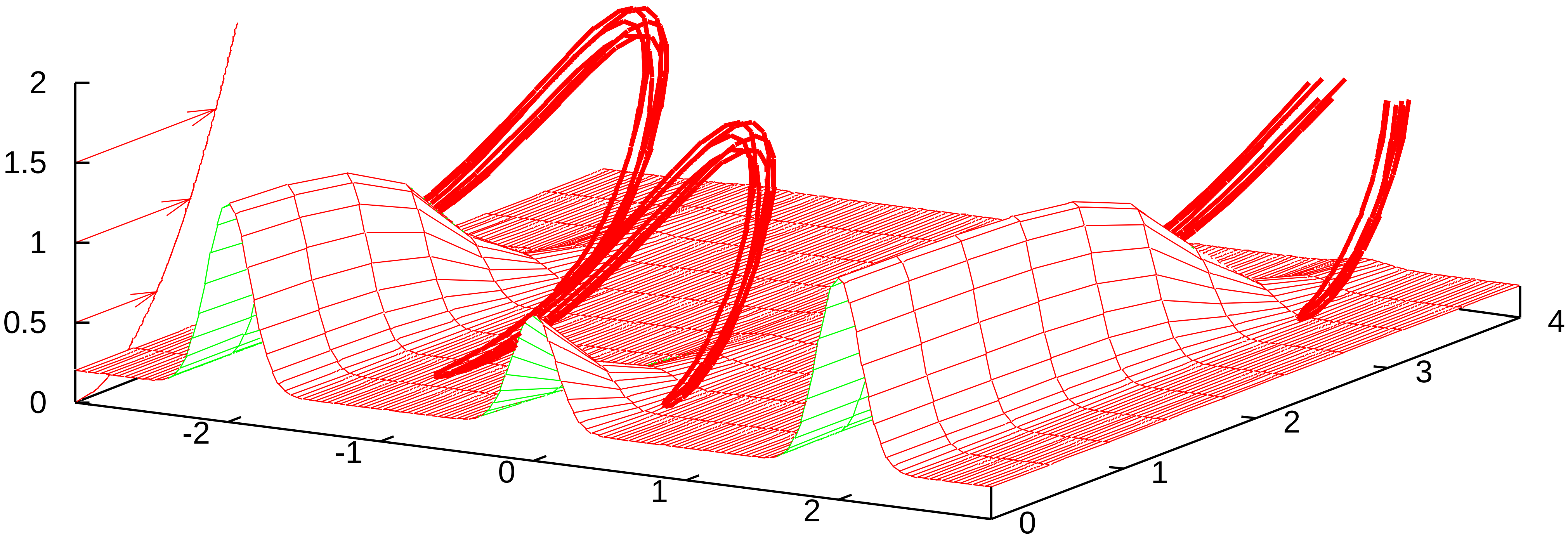
Lift-up by longitudinal vortices



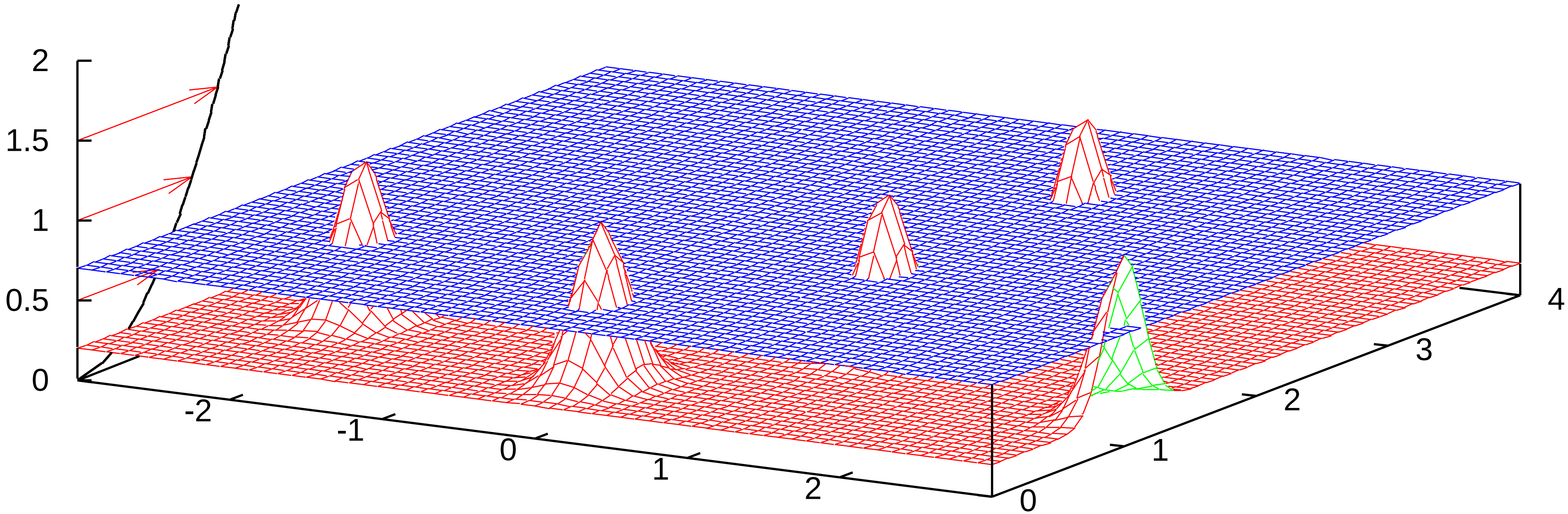
Lift-up by longitudinal vortices



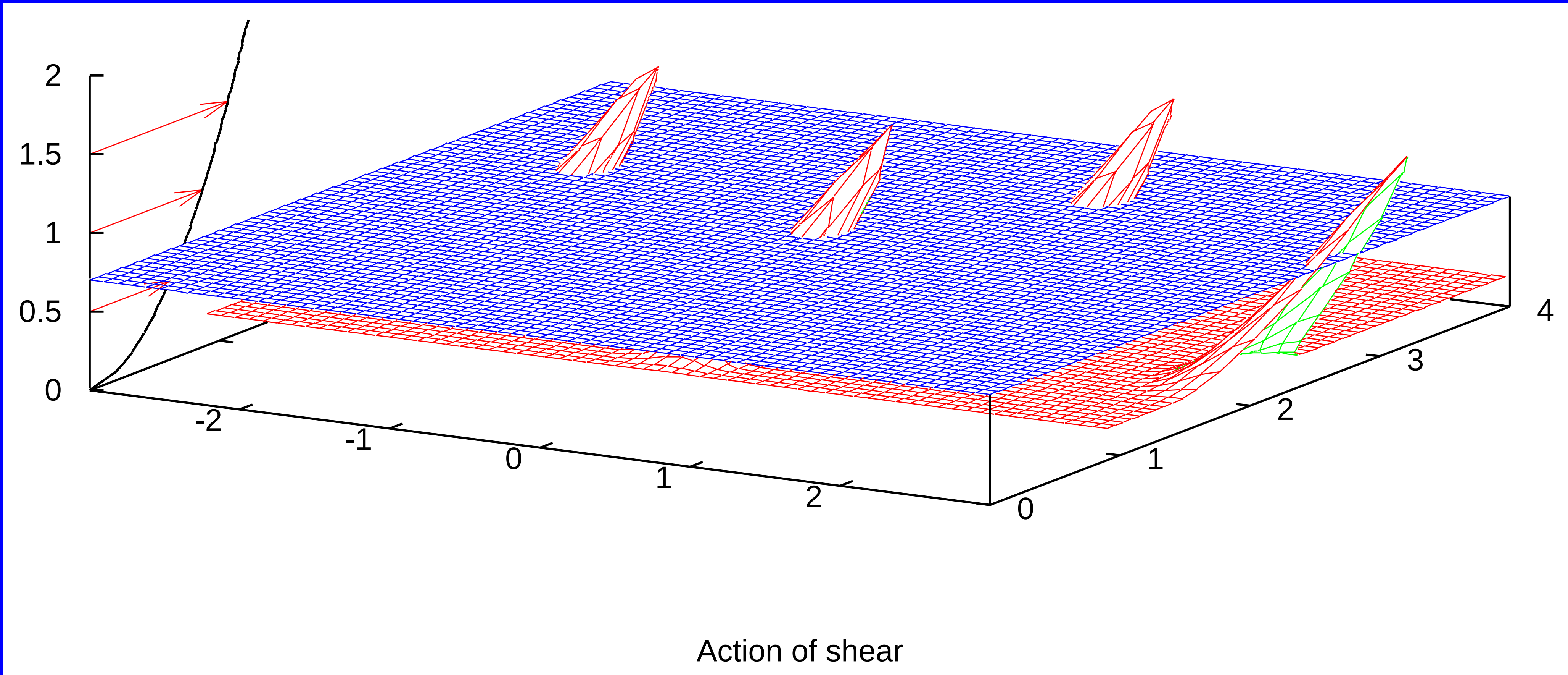
Lift-up by longitudinal vortices

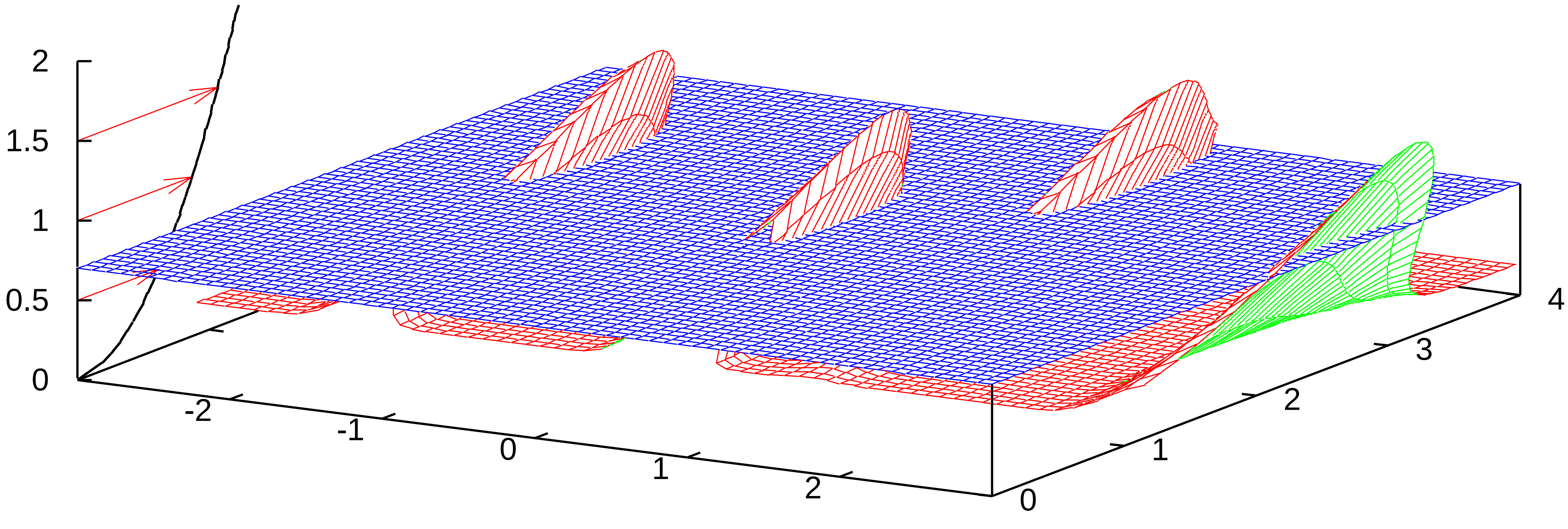


Lift-up by hairpin vortices



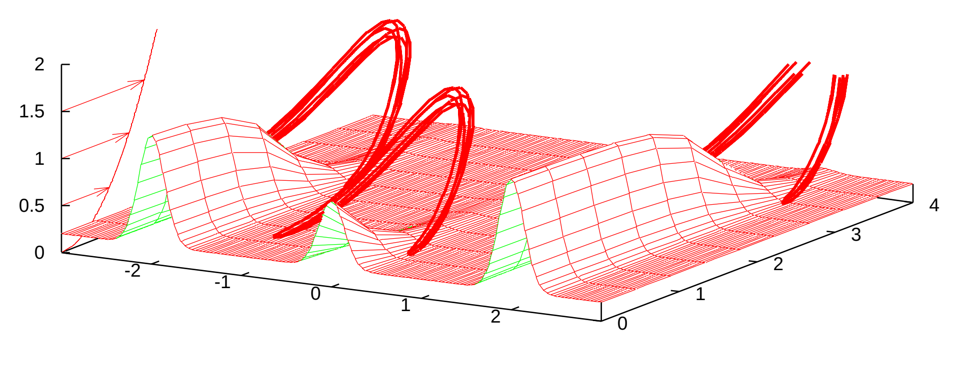
Lift-up by random wall-normal motions


2
1.5
1
0.5
0
-2
-1
0
1
2
0
1
2
3
4
Action of shear

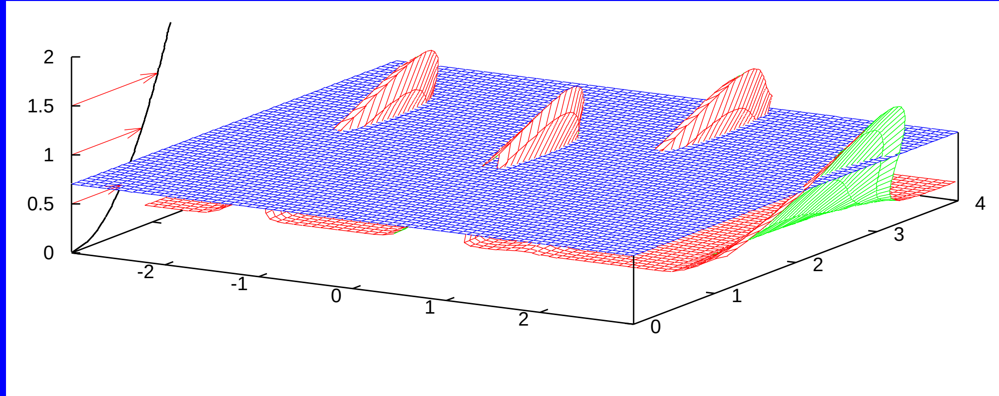


Diffusion effect

? which is true ?



Lift-up by hairpin vortices



Diffusion effect

Streaks $\Leftarrow$ organised vortices

Streaks $\Leftarrow$ properties of lift-up of the mean profile + mean shear + diffusion

Toy model:

$$\tilde{u} = L\tilde{v}, \quad \tilde{v} = N[\tilde{u}]$$

$$\tilde{u}(z) = \sum_n u_n \exp(inz), \; \tilde{v}(z) = \sum_n v_n \exp(inz).$$

L is such that

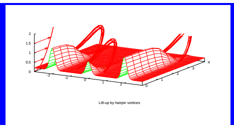
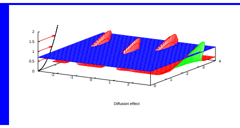
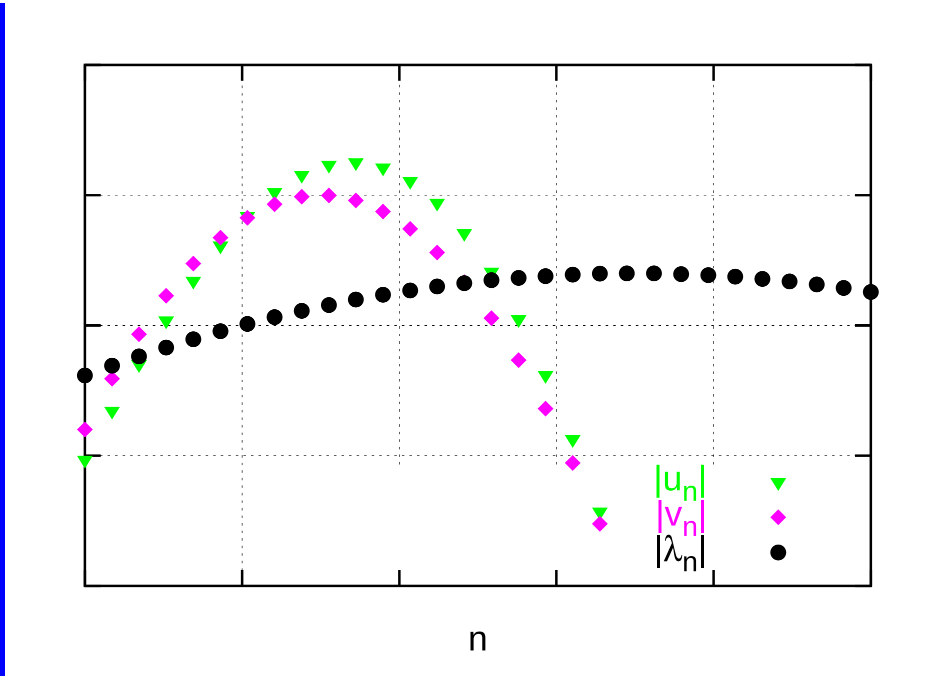
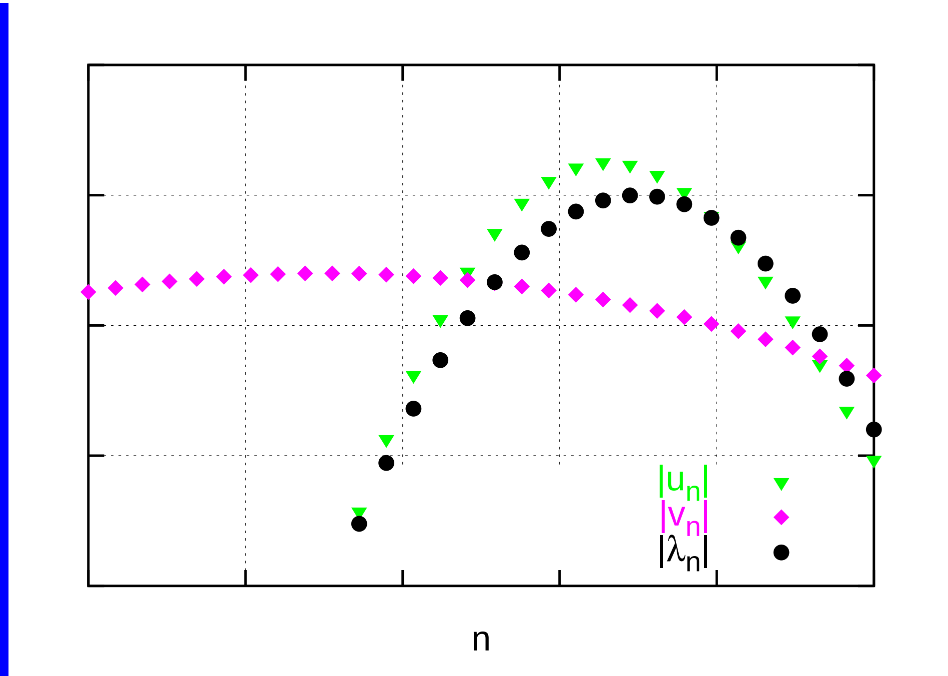
$$\boxed{u_n = \lambda_n v_n}$$

The solution has a clear pattern of $\tilde{u}(z)$: $|u_n|$ has a pronounced maximum at certain $n = k$.

Why? There are two simple explanations.

$u_n = \lambda_n v_n$

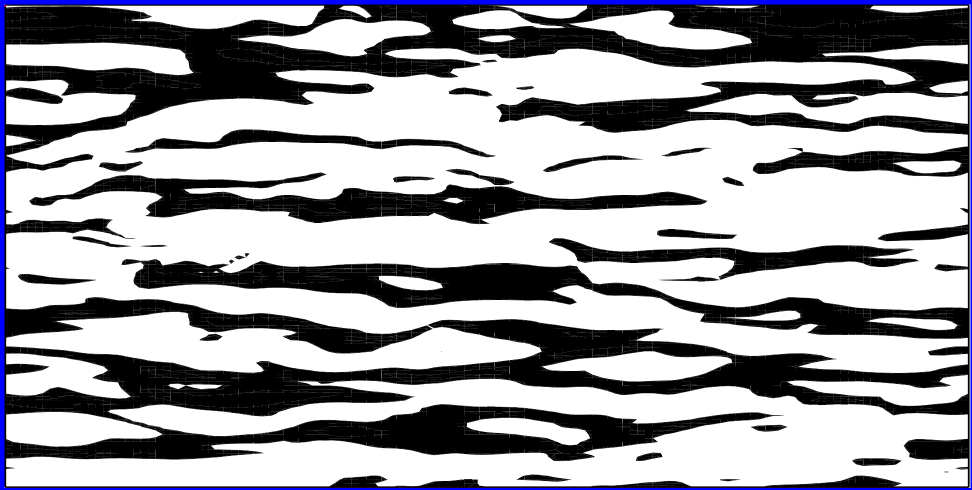
$|u_n|$ =max at $n = k$ because:

Old concept: 	New concept: 
$\vert v_n\vert$ =max at $n = k$	$\vert\lambda_n\vert$ =max at $n = k$
	
Structure of $\tilde{u} \Leftarrow$ structure of $\tilde{v}$.	Structure of $\tilde{u} \Leftarrow$ structure-forming properties of L.

? which is true ?

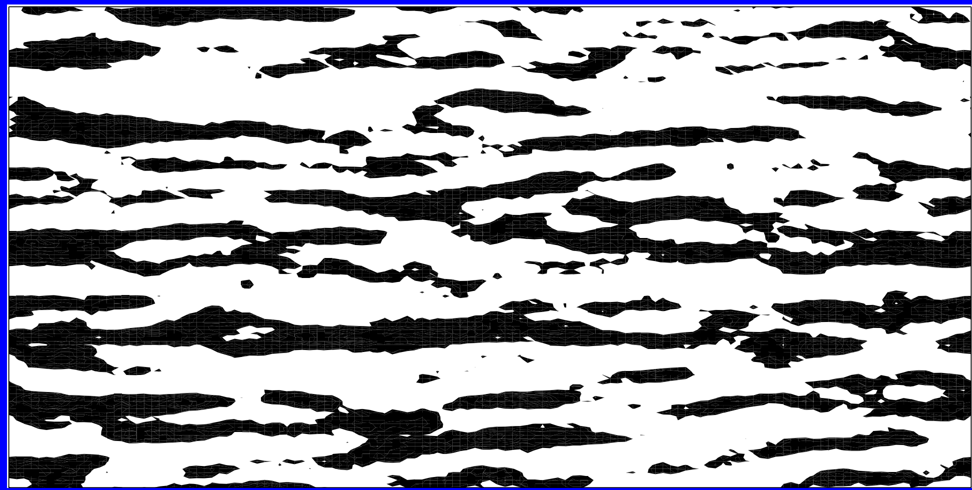
Numerical experiment-I

$$\frac{\partial c}{\partial t} + \vec{u} \cdot \nabla c = S(y) + \frac{1}{\mathrm{Re}} \nabla^2 c, \qquad \nabla \cdot \vec{u} = 0, \qquad \langle c \rangle = U(y)$$



Velocity streaks in turbulent flow.

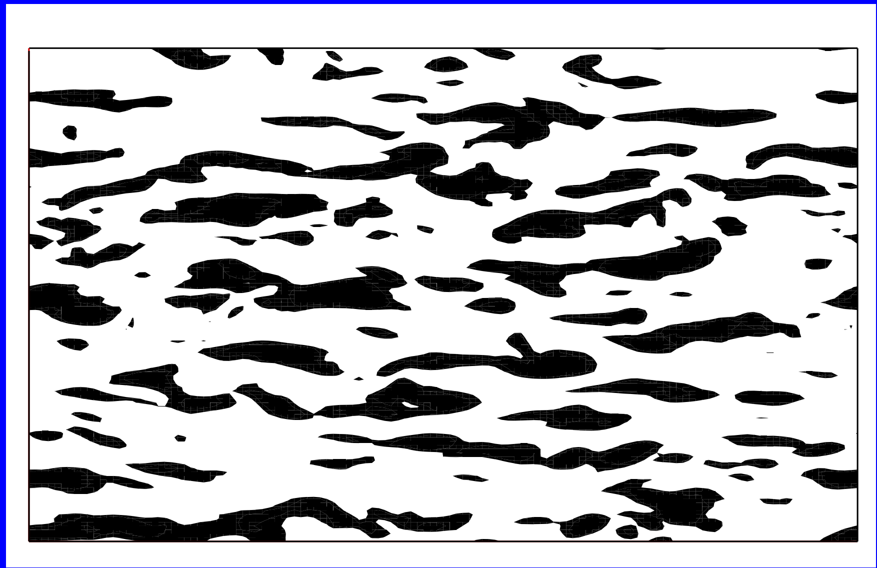
$$\frac{\partial \vec{u}}{\partial t} + \vec{u} \cdot \nabla \vec{u} = -\nabla p + \frac{1}{\mathrm{Re}} \nabla^2 \vec{u}$$



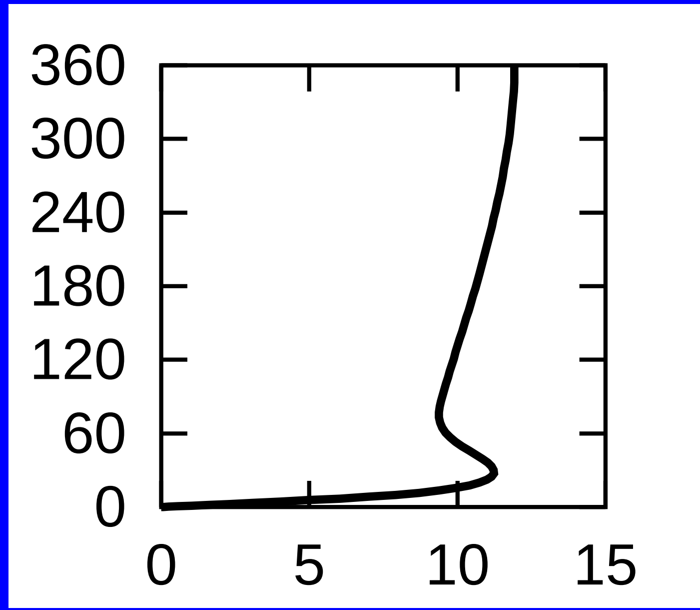
Scalar streaks in the flow with $\vec{u} = \vec{U}(y) + \nabla \phi$, where $\phi(t, x, y, z)$ is random and isotropic in planes parallel to the wall.

Numerical experiment-II: two scalars in the same flow

Scalar A

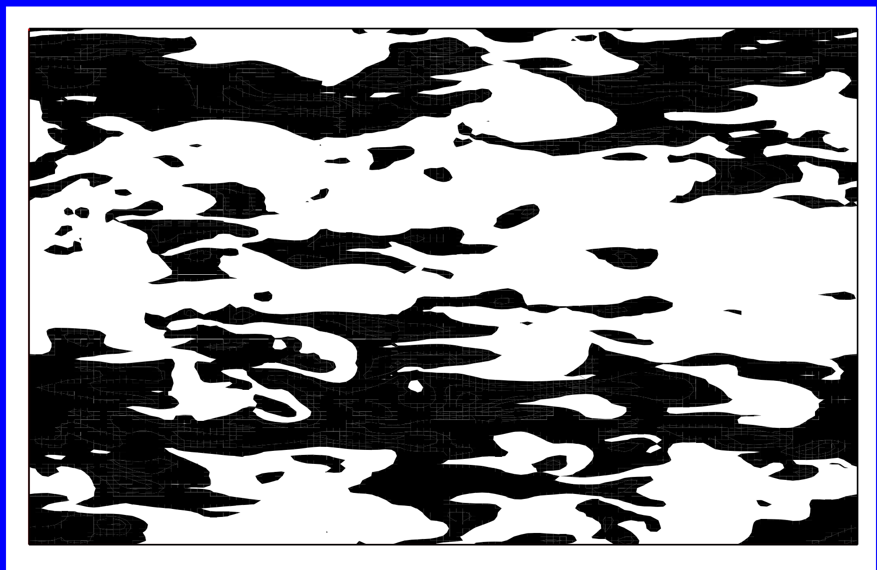


y^+

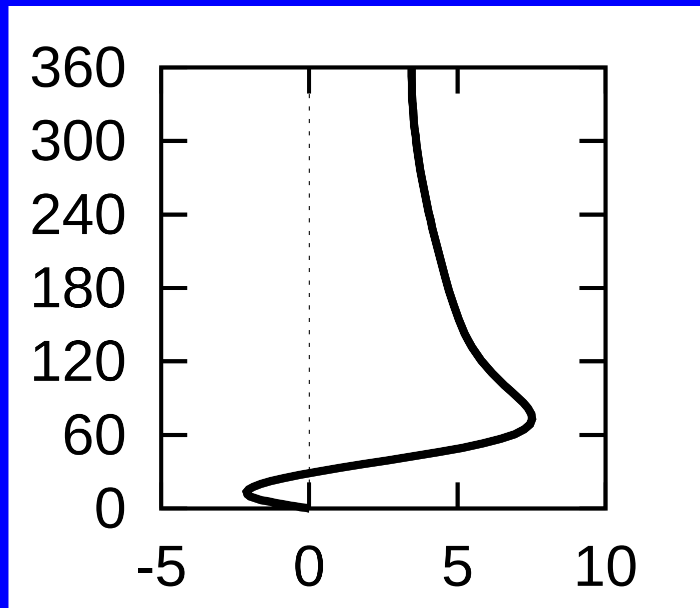


$\langle c \rangle (y)$

Scalar B



y^+



Predictive tool - the code

Input:

Gradient profile	$\frac{dU}{dy}$ or $\frac{dC}{dy}$
Reynolds number	Re
Wall-normal Reynolds stress	$\langle \hat{v}^2 \rangle (y)$
Transverse Reynolds stress	$\langle \hat{w}^2 \rangle (y)$
Visualisation plane coordinate	y_{VP}

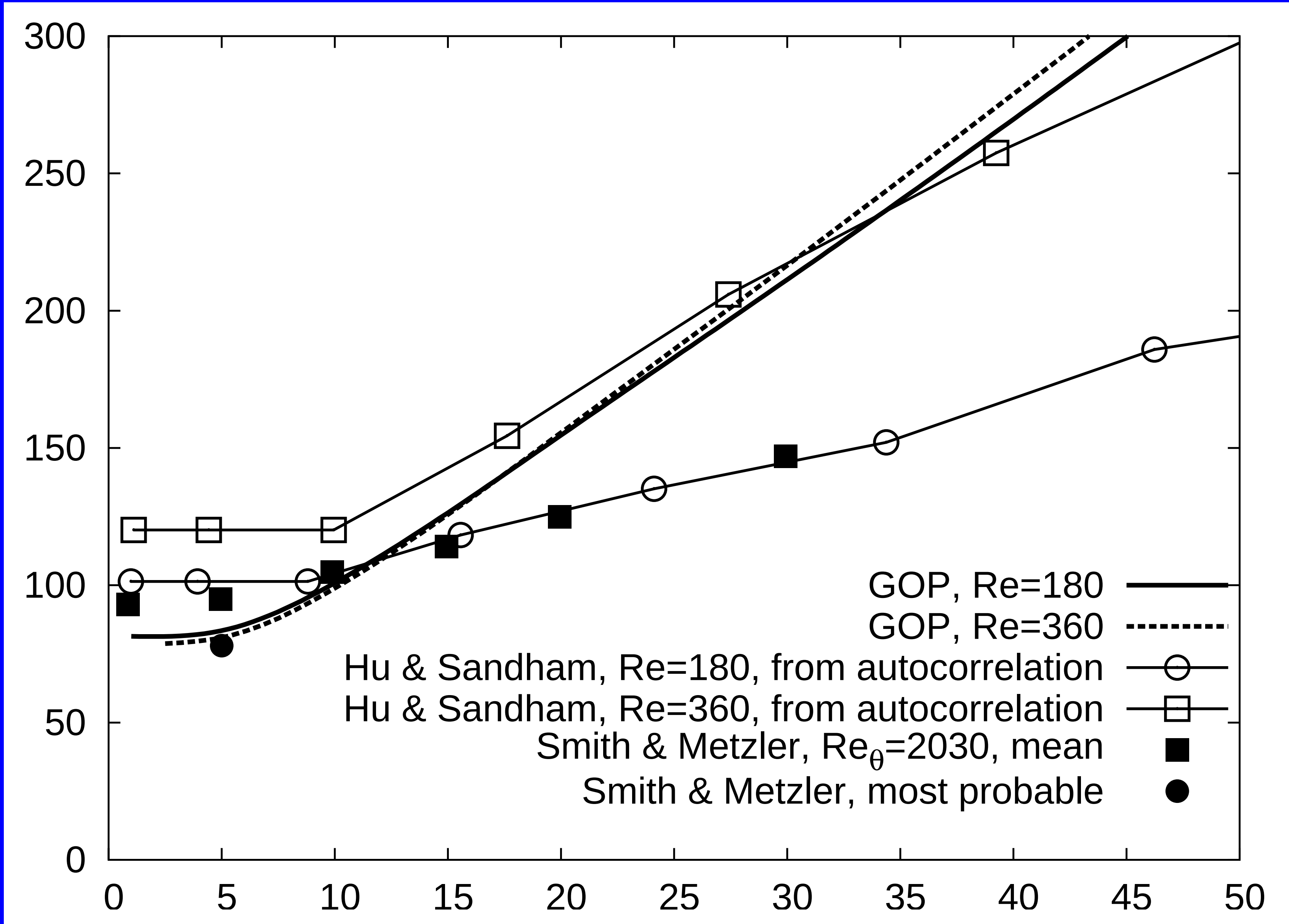
$\Downarrow$

The code

$\Downarrow$

Output: streak spacing l and some other parameters

Predictive ability of the new framework – 1

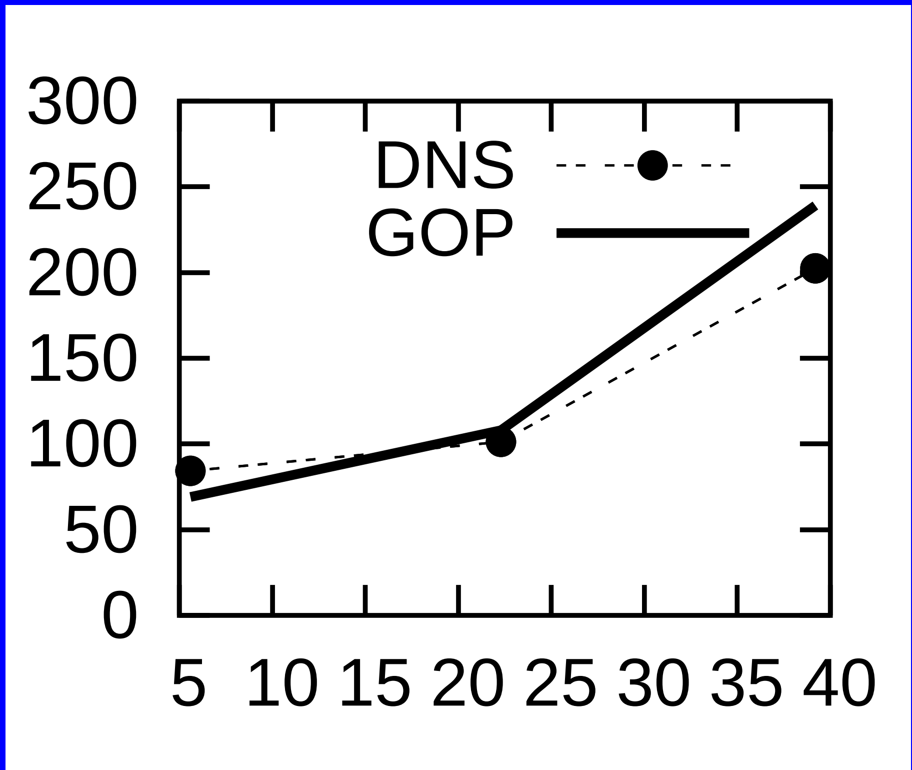


spanwise streak spacing l^+

wall – visualisation plane distance y^+

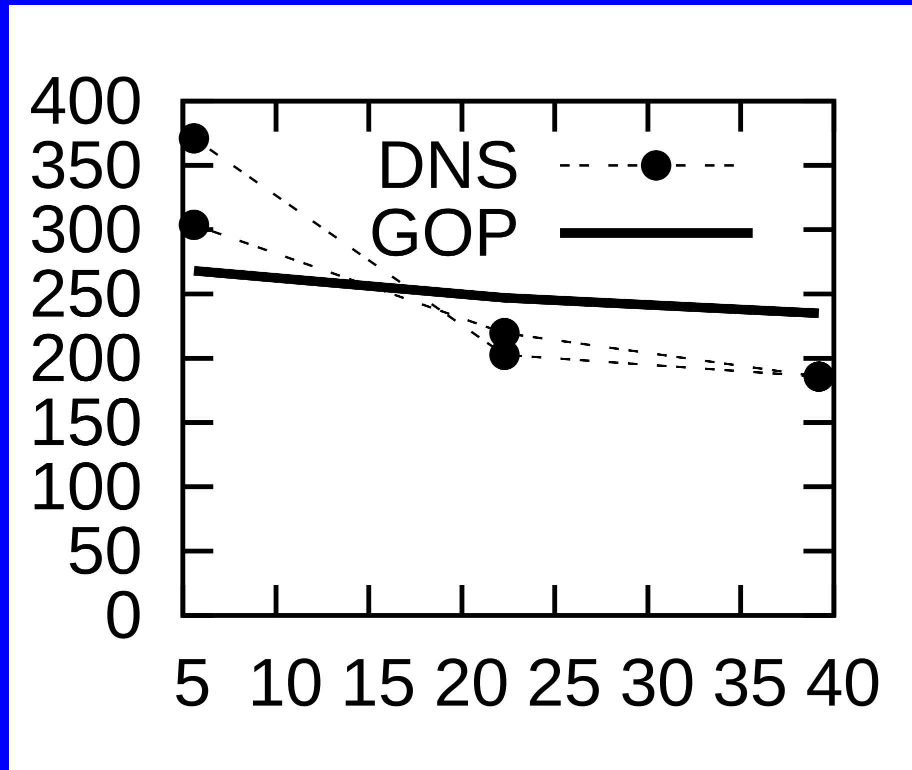
Predictive ability of the new framework – 2

l^+



y^+

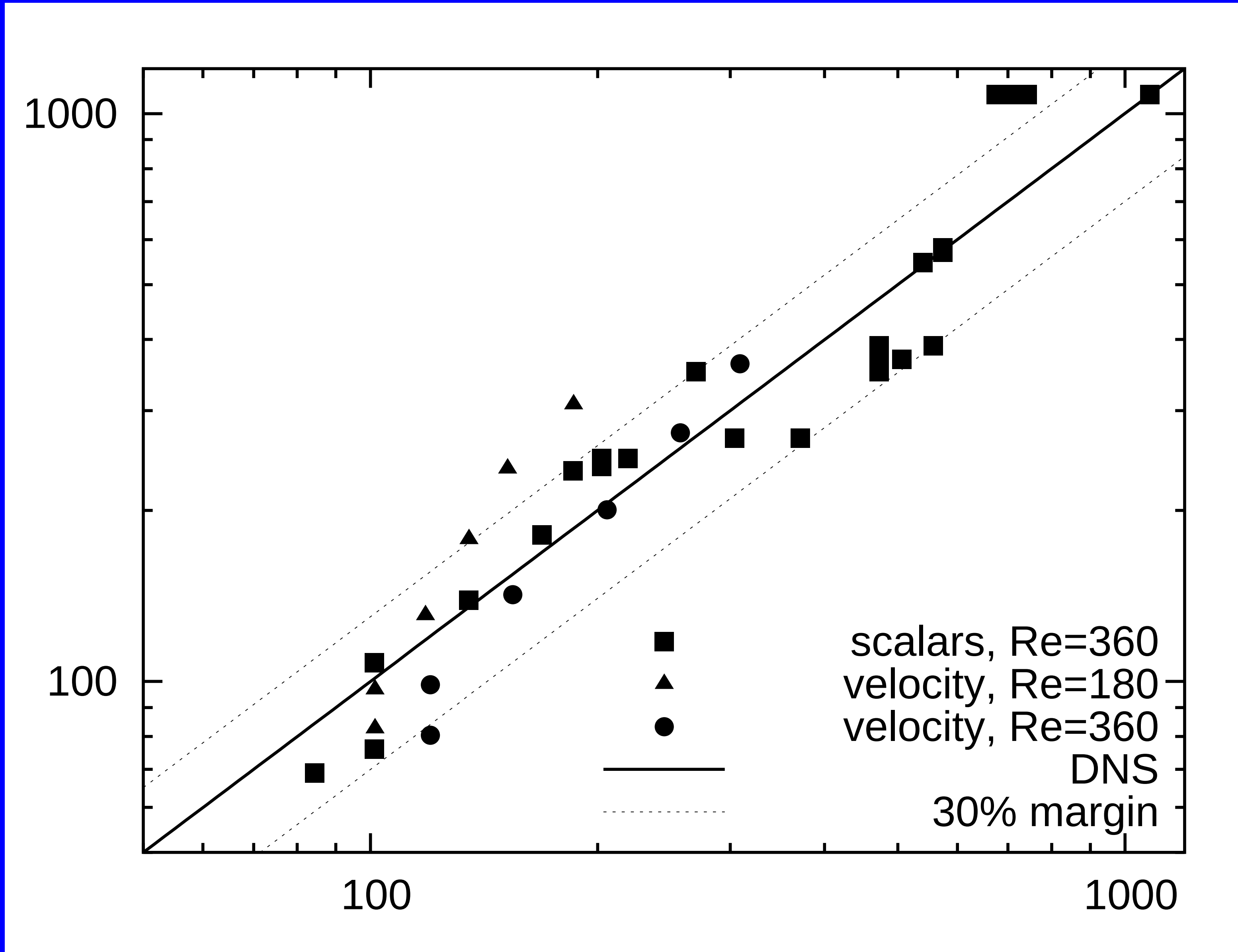
Scalar A



y^+

Scalar B

Predictive ability of the new framework – 3



l^+ as given by GOP

l^+ as given by direct numerical simulation

Implications

- New conceptual framework is easier to work in, as it is linear. Many possibilities are now open:

  - Predicting structures.
  - Riblets.
  - Active control strategies.

- Other theories (RDT, Landhal, Carpenter) based on linearised Navier-Stokes equations receive a new interpretation.

- Turbulence structure regeneration cycle/chain.

  – Let us change the conceptual framework!

Part 2. How to make predictions within the second conceptual framework? The idea:

$\tilde{u} = L\tilde{v}, \quad \tilde{v} = N[\tilde{u}].$ $\boxed{u_n = \lambda_n v_n}$

If $|u_n|$ =max at $n = k$ because $|\lambda_n|$ =max at $n = k_\lambda \approx k$ then k can be found approximately without studying the nonlinear operator N.

The optimal perturbation idea.

Let $||\tilde{w}(z)|| = \sum_n \tilde{w}_n^2$.

$$||\, L\tilde{w}\,|| \to \max, ||\tilde{w}|| = 1 \Rightarrow \tilde{w} = \tilde{v}_{\mathrm{opt}}(z) = \exp(ik_\lambda z),$$

$$\Rightarrow k \approx k_\lambda.$$

Apply the same idea to the Navier-Stokes equations:

$\vec{u}' = \vec{u} - \vec{U}$ **and** $p' = p - P$ **where** $\vec{U}$ **and** P **are the mean velocity and pressure distributions (assumed steady).**

Rewrite full NSE as

$$\overbrace{\frac{\partial \vec{u}'}{\partial t} + U \cdot \nabla \vec{u}' + \vec{u}' \cdot \nabla U + \nabla p' - \frac{1}{\mathrm{Re}} \nabla^2 \vec{u}'}^{\text{linear mechanism } (\sim L^{-1} \text{ in the model})} = \vec{F},$$

$$\vec{F} = \overbrace{-\vec{u}' \cdot \nabla \vec{u}' - \vec{U} \cdot \nabla \vec{U} - \nabla P - \frac{1}{\mathrm{Re}} \nabla^2 \vec{U}}^{(\sim N \text{ in the model})},$$

$$\nabla \vec{u}' = 0$$

Maximise $||\vec{u}'||$ **over** $\vec{F}$, $||\vec{F}|| = 1$.

Simplification: consider $\vec{F} = \delta(t)\vec{u_0}(\vec{x})$ **only.**

In 1993 Butler and Farrell solved the so-called optimal perturbation problem for a flow in a plane channel:

$$\frac{\partial \vec{u}'}{\partial t} + U \cdot \nabla \vec{u}' + \vec{u}' \cdot \nabla U + \nabla p' - \frac{1}{\mathrm{Re}} \nabla^2 \vec{u}' = 0$$

$$\nabla \vec{u}' = 0, \quad \vec{u}'_{|t=0} = \vec{u}'_0$$

Find $\vec{u}'_0$ such that $||\vec{u}'_0|| = 1$ and the $\max_{t>0} ||\vec{u}'||$ is the greatest possible.

This gave streaks with spacing $l^+ = 540$. Assuming that the perturbation exists only for $t < t_e = q^2/\epsilon = \frac{\langle u_i u_i \rangle}{\langle \nu u_{i,j} u_{i,j} \rangle}$ gave $l^+ \approx 100$ in agreement with near-wall measurements.

Butler and Farrell did not explain why optimal perturbations were expected to give predictions about the developed turbulent flow.

Toy model extended: $\tilde{u} = L\tilde{v}, \quad \tilde{v} = N[\tilde{u}, \tilde{c}], \quad \tilde{c} = M\tilde{v}.$

$$\boxed{\tilde{u}_n = \lambda_n \tilde{v}_n} \qquad \boxed{\tilde{c}_n = \mu_n \tilde{v}_n}$$

$\lambda_n = $max for $n = k$ $\qquad$ $\mu_n =$max for $n = l \neq k$.

$$||\, L\tilde{w}\, || \rightarrow \text{max},\ ||\tilde{w}|| = 1 \Rightarrow \tilde{w} = \tilde{v}_{\text{opt}}(z) = \exp(ikz),\ \Rightarrow \text{k}.$$

$$||M\tilde{w}|| \rightarrow \text{max},\ ||\tilde{w}|| = 1 \Rightarrow \tilde{w} = \tilde{v}^c_{\text{opt}}(z) = \exp(ilz),\ \Rightarrow \text{l}.$$

$$\tilde{v} \not\approx \tilde{v}_{\text{opt}}(z) \text{ and } \tilde{v} \not\approx \tilde{v}^c_{\text{opt}}(z)$$

1. Optimal perturbation is not a model of real flow $\Rightarrow$ no reason to limit its lifetime.

2. Maximising $||u = L\tilde{w}||$ does not give the dominant wavenumber of $\tilde{c}$ and vice versa $\Rightarrow$ choose your norms properly.

GOP: approximate simplified solution – 1.

$$\begin{aligned} ||u'||^2_{y_{VP}} &= \max_t \int_{\mathrm{VP}} {u'}^2\, dx\, dz \to \max, (\text{or } ||c'||^2_{y_{VP}} \to \max), \\ ||\vec{u}_0||_i &= \int \left(\frac{v_0^2}{\langle \hat{v}^2 \rangle} + \frac{w_0^2}{\langle \hat{w}^2 \rangle} \right) dx\, dy\, dz = 1 \end{aligned} \qquad (*)$$

VP is the visualisation plane at $y = y_{VP}$ from the wall, $\langle \hat{v}^2 \rangle(y)$ and $\langle \hat{w}^2 \rangle(y)$ are the normal Reynolds stresses, and u' and c' satisfy

$$\begin{aligned} \frac{\partial c'}{\partial t} + v' \frac{dC}{dy} &- \frac{1}{\mathrm{Re}} \nabla^2 c' = 0 \\ \frac{\partial u'}{\partial t} + v' \frac{dU}{dy} &- \frac{1}{\mathrm{Re}} \nabla^2 u' = 0 \\ \frac{\partial v'}{\partial t} + \frac{\partial p'}{\partial y} &- \frac{1}{\mathrm{Re}} \nabla^2 v' = \delta(t) v_0(y,z) \\ \frac{\partial w'}{\partial t} + \frac{\partial p'}{\partial z} &- \frac{1}{\mathrm{Re}} \nabla^2 w' = \delta(t) w_0(y,z) \\ \frac{\partial v'}{\partial y} + \frac{\partial w'}{\partial z} &= 0 \end{aligned}$$

GOP: approximate simplified solution – 2.

In Fourier space $u' = iu_\beta e^{i\beta z}$, $v' = iv_\beta e^{i\beta z}$ and $w' = w_\beta e^{i\beta z}$.

$$u_\beta(t,y) = \int (G_v(t,\beta,y,\eta)v_{\beta 0}(\eta) + G_w(t,\beta,y,\eta)w_{\beta 0}(\eta))\, d\eta \quad (**)$$

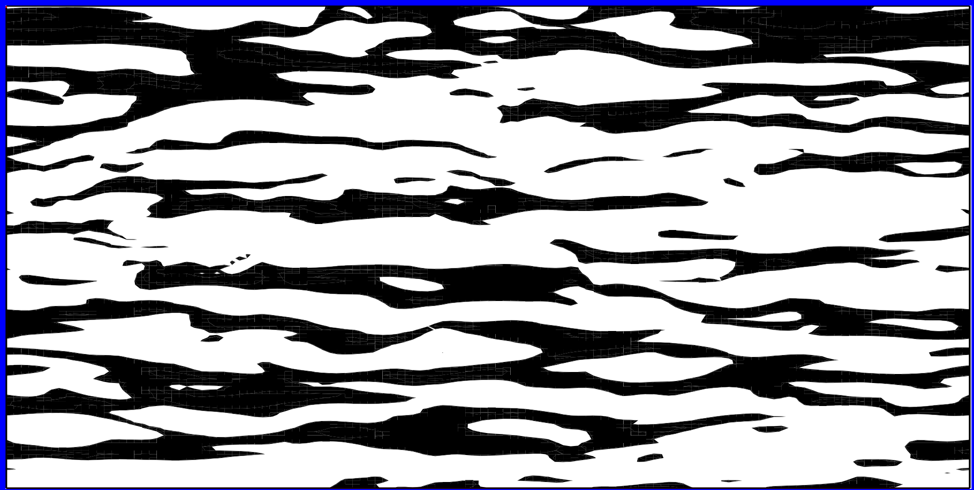
$$\langle u_\beta^2 \rangle \sim \int \Big(G_v^2(t,\beta,y,\eta)\langle v_\beta^2(\eta)\rangle + G_w^2(t,\beta,y,\eta)\langle w_\beta^2(\eta)\rangle \Big)\, d\eta.$$

$$\int \left(G_v^2(t,\beta,y,\eta)\langle \hat{v}^2(\eta)\rangle + G_w^2(t,\beta,y,\eta)\langle \hat{w}^2(\eta)\rangle \right)\, d\eta \to \max$$

is equivalent to solving (*) using (**) but without continuity imposed on v, w.

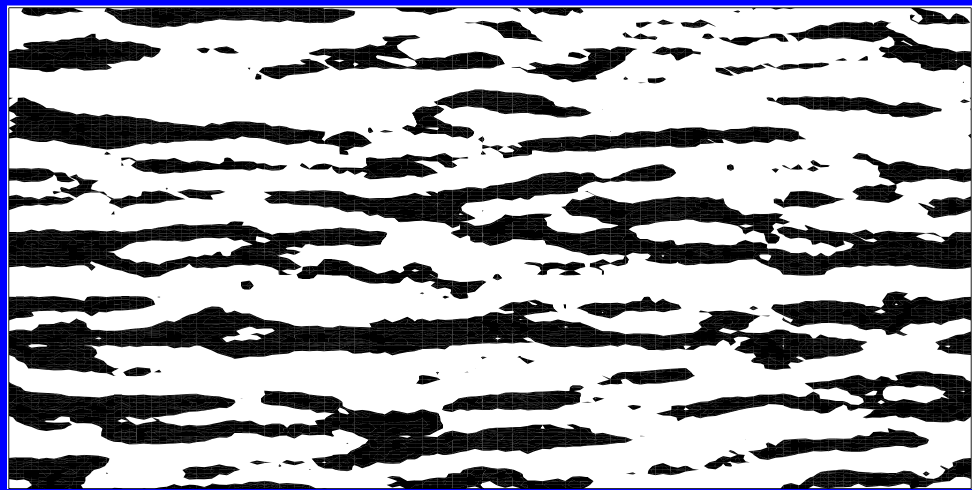
Numerical experiment-I revisited

$$\frac{\partial c}{\partial t} + \vec{u} \cdot \nabla c = S(y) + \frac{1}{\text{Re}} \nabla^2 c, \qquad \nabla \cdot \vec{u} = 0, \qquad \langle c \rangle = U(y)$$



Velocity streaks in turbulent flow.

$$\frac{\partial \vec{u}}{\partial t} + \vec{u} \cdot \nabla \vec{u} = -\nabla p + \frac{1}{\text{Re}} \nabla^2 \vec{u}$$



Scalar streaks in the flow with $\vec{u} = \vec{U}(y) + \nabla \phi$, where $\phi(t, x, y, z)$ is random and isotropic in planes parallel to the wall.

Simplified GOP (with J.Weller).

$$\frac{\partial c'}{\partial t} + v'\frac{dC}{dy} - \frac{1}{\mathrm{Re}}\nabla^2 c' = 0 \ ,$$

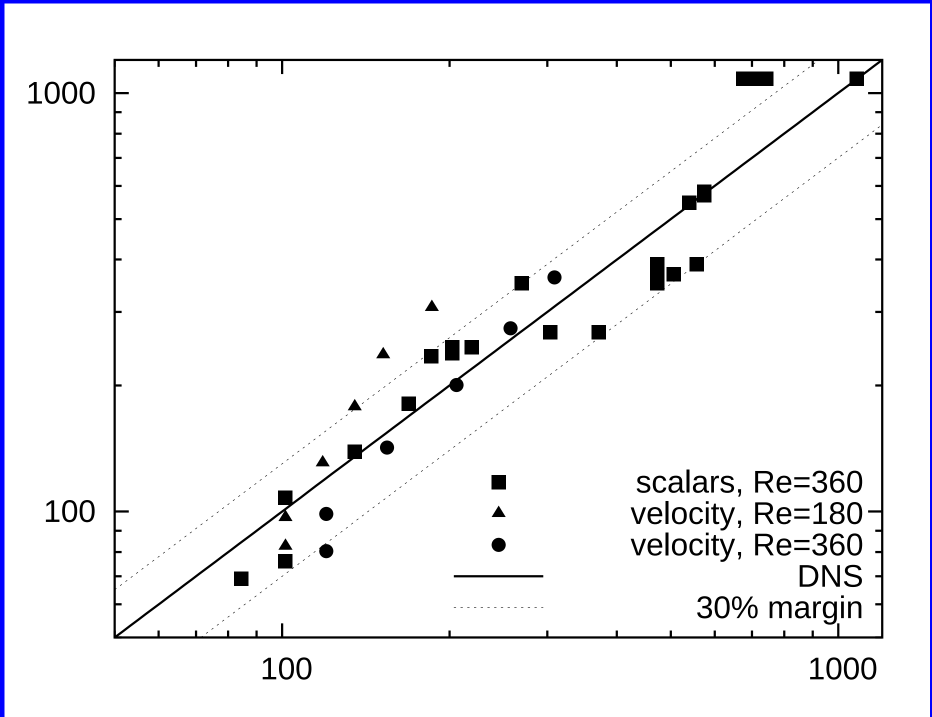
$$\frac{\partial v'}{\partial y} + \frac{\partial w'}{\partial z} = 0,$$

$$v', w', c' \sim \exp(i\omega t)$$

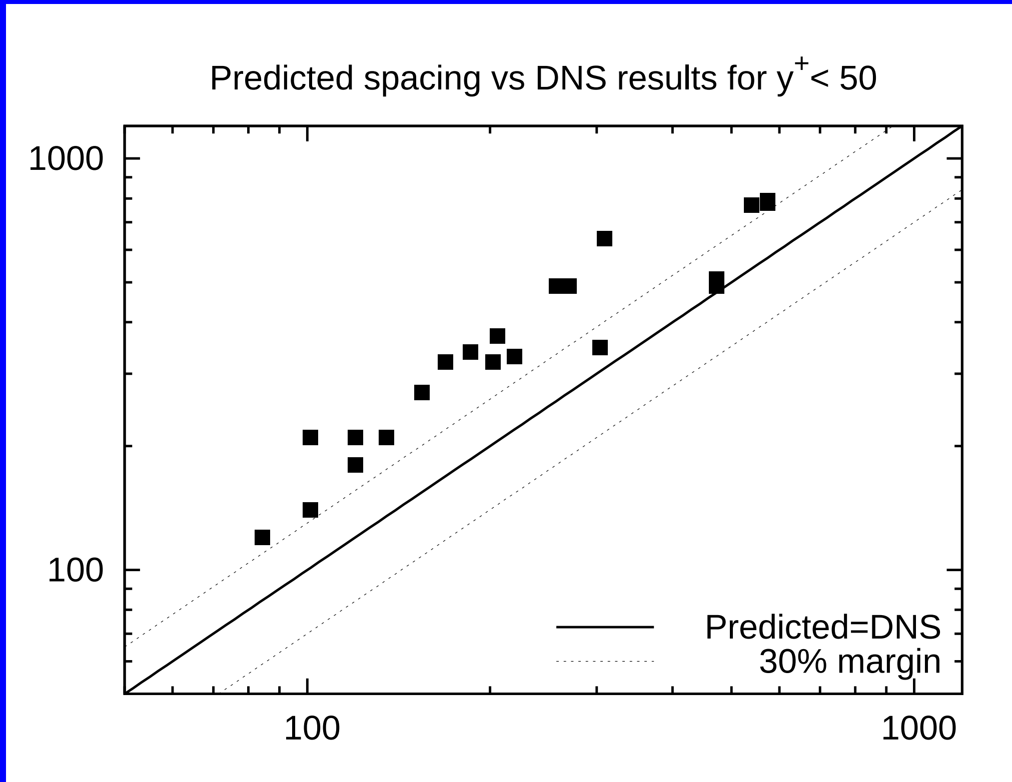
$$||c'||_{y_{VP}} \to \max,$$

$$||v'||_i = \int \left(\frac{v'^2}{\langle \hat{v}^2 \rangle} + \frac{w'^2}{\langle \hat{w}^2 \rangle} \right) dx\, dy\, dz\, dt = 1$$

Two GOPs results



Original GOP



Simplified GOP

GOP: approximate simplified solution – 2 revisited.

In Fourier space $u' = iu_\beta e^{i\beta z}$, $v' = iv_\beta e^{i\beta z}$ and $w' = w_\beta e^{i\beta z}$.

$$u_\beta(t,y) = \int \left(G_v(t,\beta,y,\eta)v_{\beta 0}(\eta) + G_w(t,\beta,y,\eta)w_{\beta 0}(\eta)\right) d\eta \quad (**)$$

$$\langle u_\beta^2 \rangle \sim \int \Big(G_v^2(t,\beta,y,\eta)\langle v_\beta^2(\eta)\rangle + G_w^2(t,\beta,y,\eta)\langle w_\beta^2(\eta)\rangle \Big)\, d\eta.$$

$$\int \left(G_v^2(t,\beta,y,\eta)\langle \hat{v}^2(\eta)\rangle + G_w^2(t,\beta,y,\eta)\langle \hat{w}^2(\eta)\rangle \right) d\eta \rightarrow \max$$

is equivalent to solving (*) using (**) but without continuity imposed on v, w.

The difficulty:

$$\tilde{u} = L\tilde{v}, \quad \tilde{v} = N[\tilde{u}]$$

$$\tilde{u}(z) = \sum_n u_n \exp(inz),\ \tilde{v}(z) = \sum_n v_n \exp(inz).$$

$$u_n = \lambda_n v_n,\ v_n = N_n[\tilde{u}]$$

But! The same system can be rewritten as

$$u_n = \lambda_n a_n w_n,\ w_n = a_n^{-1} N_n[\tilde{u}]$$

or $\tilde{u} = LA\tilde{w}_A,\ \tilde{w}_A = A^{-1}N[\tilde{u}]$

L **is not unique!**

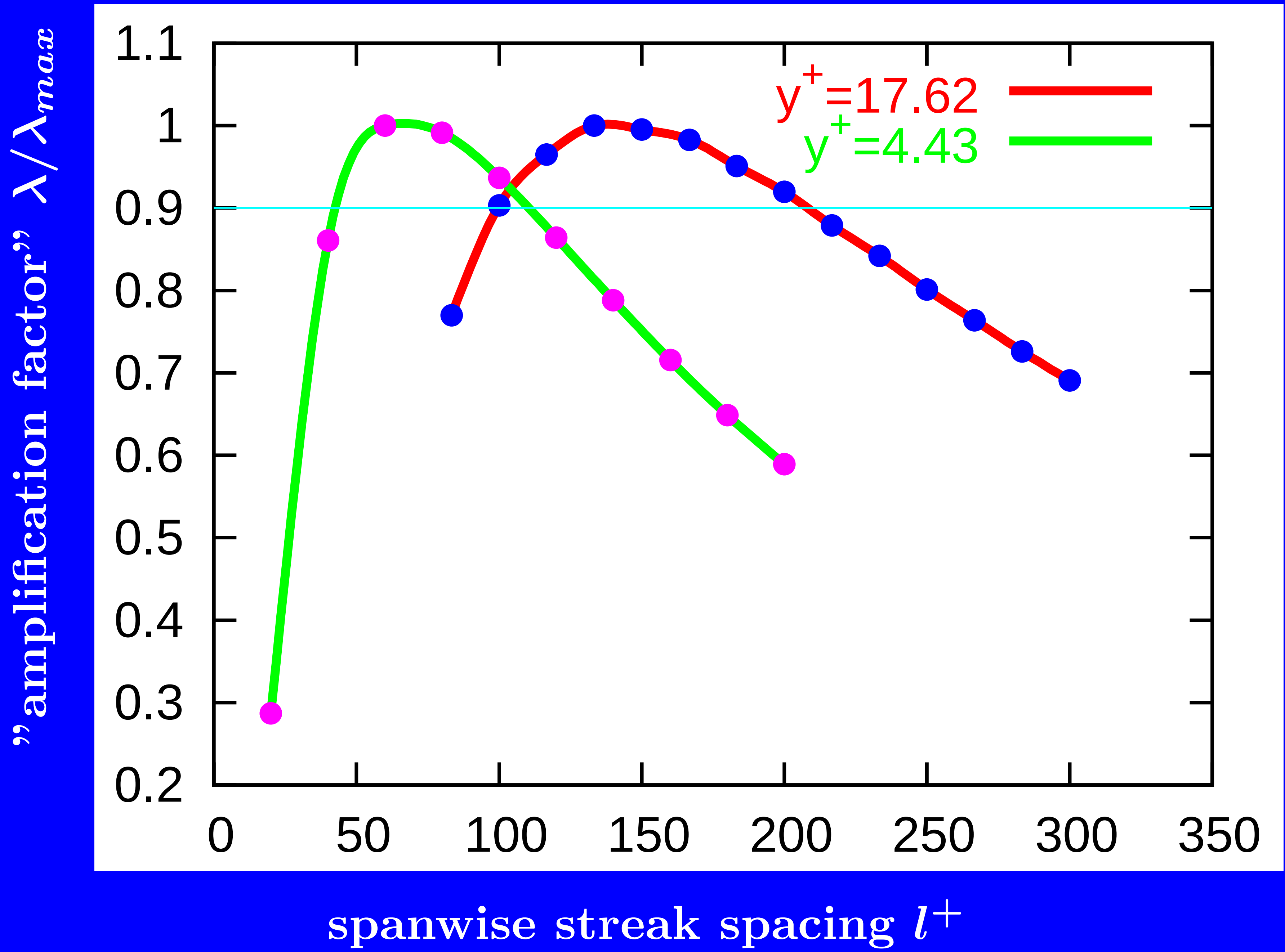
The ideas of how to overcome this difficulty are left to your imagination :-).

Conclusions

- **Streak structure is dictated mostly by the structure-forming properties of the combination of lift-up of the mean profile, mean shear, and viscous diffusion and only to a far lesser extent by the structure of the wall-normal motions.**
- **The approximate approach (GOP) based on this idea has a significant predictive ability.**
- **The new conceptual framework is a wide field for further research, both for theoretical justification of the approach and in applying the approach to other problems.**

Extras

GOP selectivity



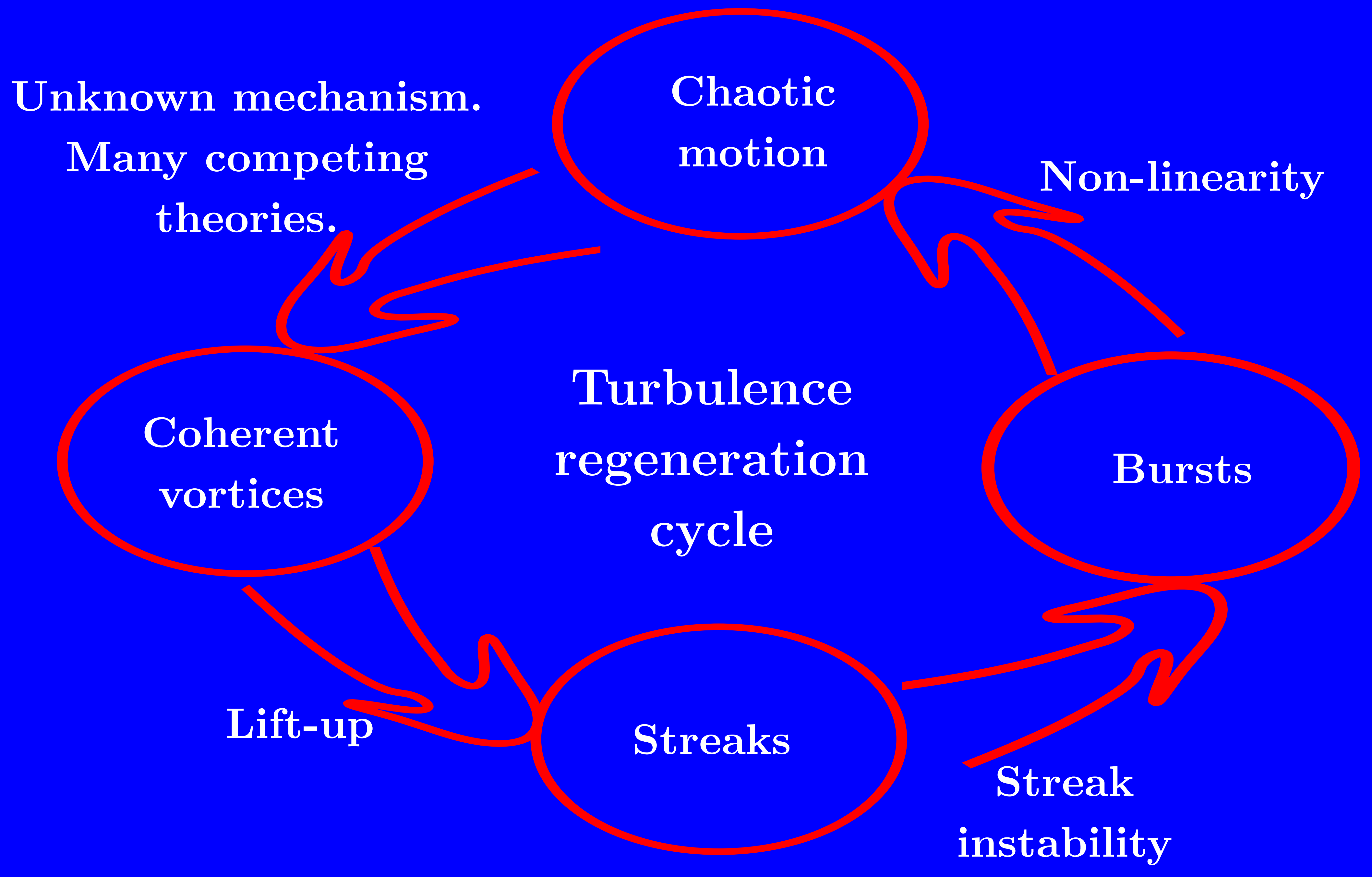
The works within the second conceptual framework include:

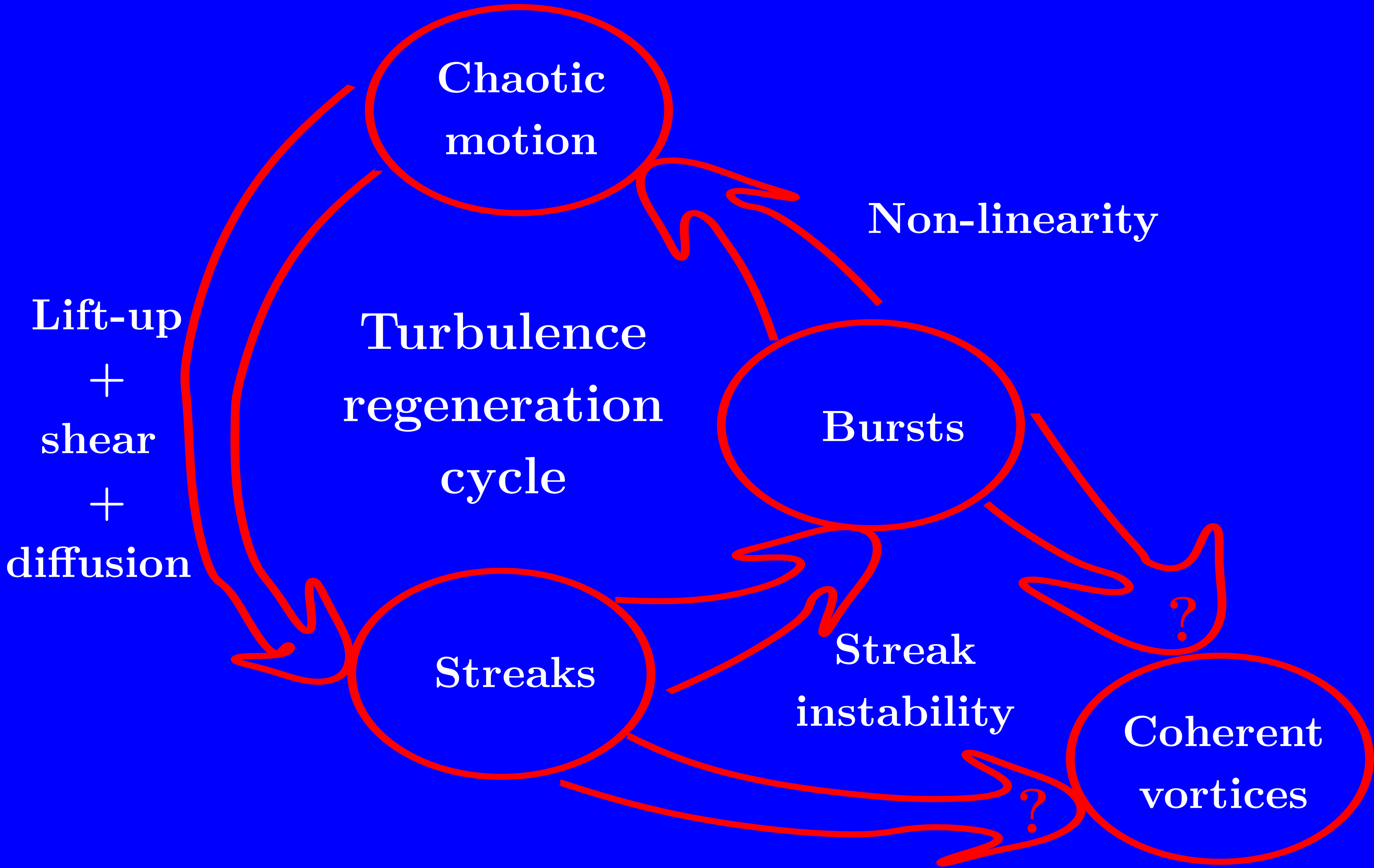
Rapid Distortion Theory (RDT): Batchelor and Proudman 1954, Moffat 1965, Hunt 1973, Goldstein and Durbin 1980, Cambon and Scott 1999, Nazarenko, Kevlahan and Dubrulle 1999 and many more.

Landahl, M. T. 1989 Boundary layer turbulence regarded as a driven linear system. *Physica D* 37, 11–19.

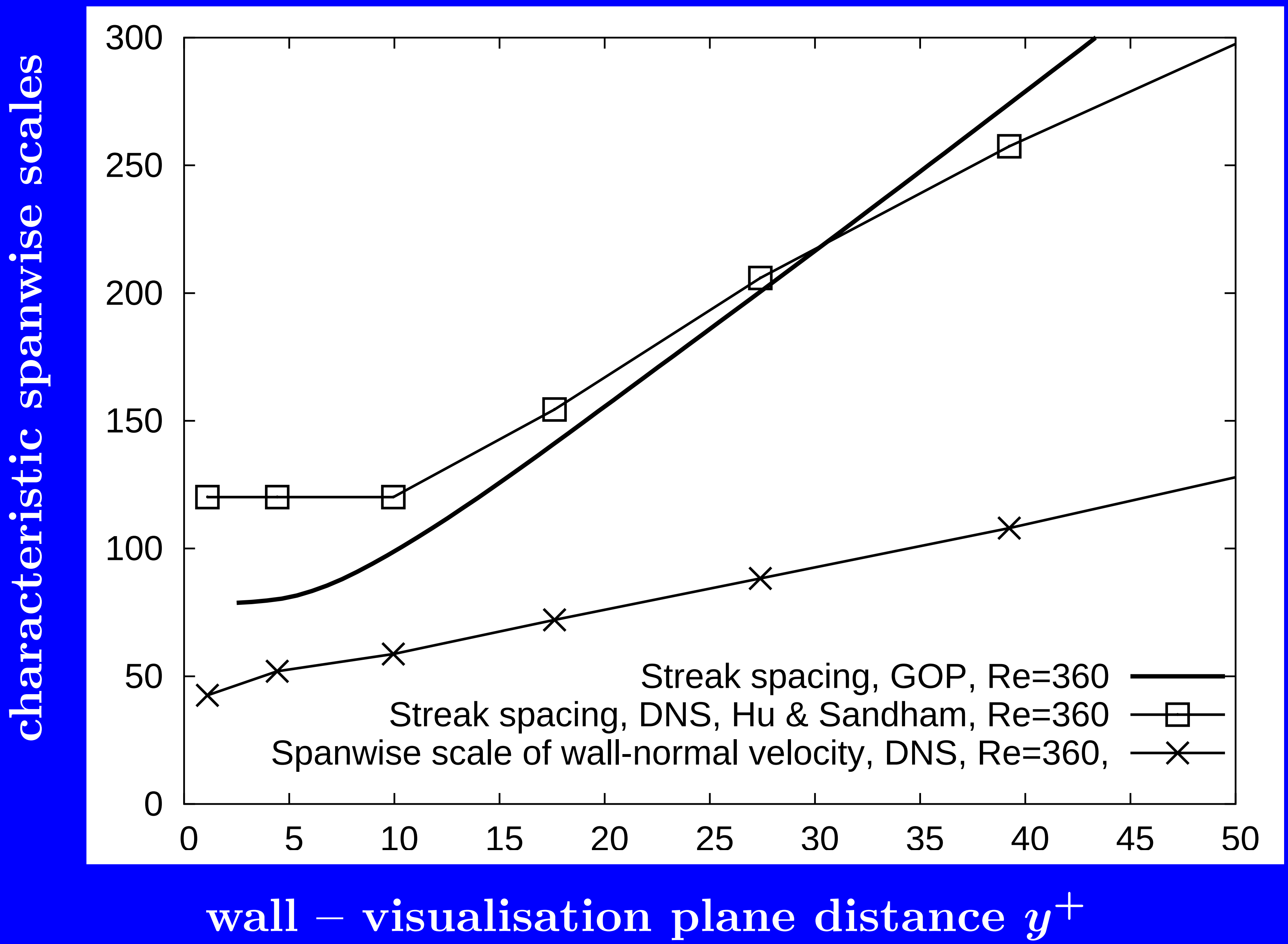
Butler, K. M. & Farrell, B. F. 1993 Optimal perturbations and streak spacing in wall-bounded turbulent shear flows. *Phys. of Fluids A* 5, 774–777.

Carpenter, P., Ali, R., Davies, C. & Lockerby, D. 2003 A simple computational model for studying the control of viscous sublayers. In *5th Euromech Fluid Mechanics Conference, Toulouse, 24-28 August*, p. 367.

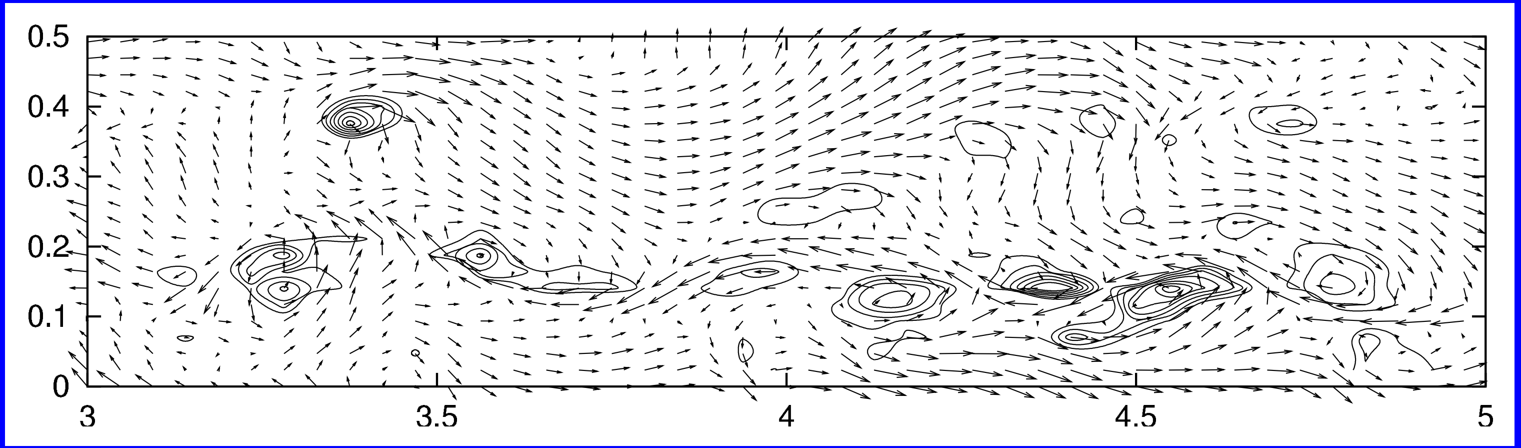

Unknown mechanism.
Many competing
theories.
Chaotic
motion
Non-linearity
Coherent
vortices
Turbulence
regeneration
cycle
Bursts
Lift-up
Streaks
Streak
instability


Chaotic motion
Non-linearity
Lift-up + shear + diffusion
Turbulence regeneration cycle
Bursts
Streaks
?
Streak instability
Coherent vortices
?

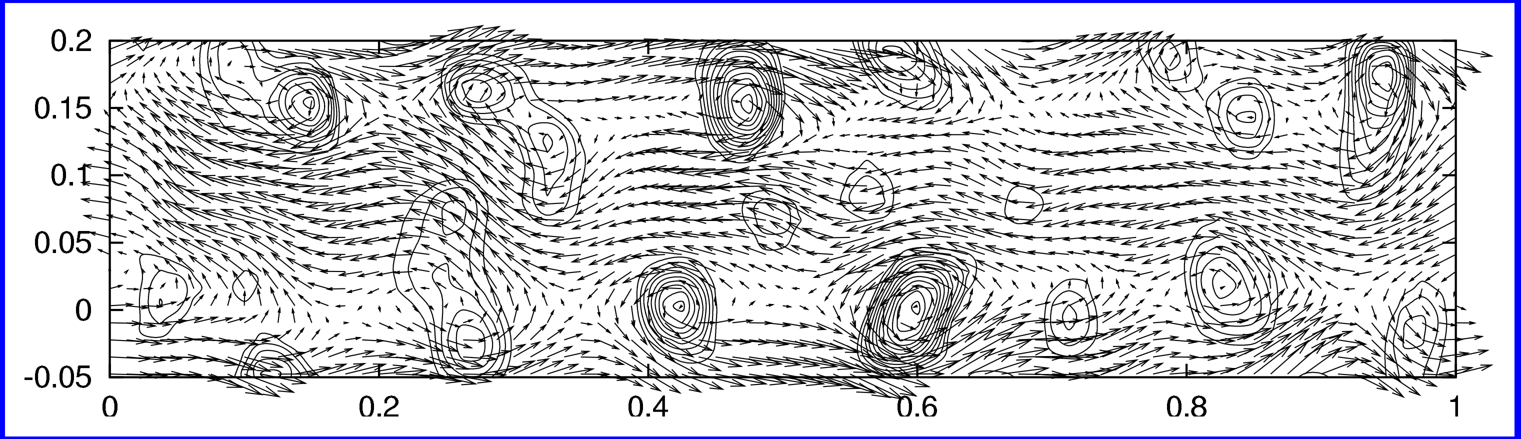
Spanwise scales comparison



Is there any relation at all between vortices and streaks?



DNS by Hu and Sandham, with Galilean decomposition and swirling strength as in Tomkins & Adrian, JFM, 2003, v. 490.



Placebo field=organised streaks + random field, with Galilean decomposition and swirling strength.

Thanks to A. Iollo (Université Bordeaux 1), G. M. Di Cicca (Politecnico di Torino), Z. W. Hu, N. D. Sandham (University of Southampton), and A.V. Smirnov (West Viriginia University) who provided codes and data.