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# Communication with reentry space vehicles via short pulses

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**Abstract.** The resonant absorption of a short pulse of electromagnetic radiation in a plasma slab is considered in order to examine the effectiveness of using short pulses for information transfer through the plasma sheaths surrounding reentry vehicles. The amplitudes of the Raman scattering (RS) on the Langmuir oscillations associated with resonant absorption and the amplitude of the Brillouin scattering (BS) on the nonlinear modifications of the plasma profile are calculated. On-board detection of the RS and BS signals are proposed as parallel and independent channels for reading the message encoded on the electromagnetic pulse sequence. The information transfer capacity of two other nonlinear effects, the generation of the fast electrons in the wave breaking process, and the excitation of the ion-sound waves by the radiative pressure of the Langmuir oscillations are also considered.

## Introduction

A space vehicle, reentering the atmosphere at high Mach number, is surrounded by a plasma sheath reflecting radio waves. Communication through the plasma sheath during the "black-out" has long been a subject of investigation. Many schemes have been suggested, including the strong magnetic field generation to modify plasma dispersion properties and chemical seeding to reduce the ionization level. Nevertheless, to date, no cheap and reliable method of communication with the reentry vehicles has been found.

Recently, a new approach to the problem was suggested [*Nazarenko et al.*, 1994]. This approach is based on the following idea, illustrated in Figure 1. A p-polarized electromagnetic wave, obliquely incident to the plasma sheath, penetrates the latter to the point of resonant absorption where the local plasma frequency matches the frequency of the incident signal. As a result of the resonant transformation, a buildup of a narrow peak of Langmuir oscillations takes place, with the electric field intensity greatly exceeding the intensity of the incident electromagnetic wave. A probe electromagnetic beam with frequency above the maximal plasma frequency of the plasma sheath, launched from the vehicle itself, is scattered on these Langmuir oscillations, a process called Raman scattering (RS). The scattered Stokes wave returns to the vehicle carrying with it the information on the signal wave. In this way, using nonlinear interactions, one can penetrate the plasma sheath by employing incident signals with relatively low frequency levels chosen to optimize propagation through the atmosphere and to take advantage of more powerful generators.

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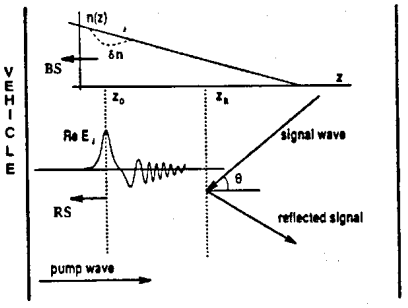


**Figure 1.** The communication scheme using Raman scattering (RS) and Brillouin scattering (BS) for information transfer. Here $n(z)$ is the unperturbed profile of the plasma concentration, $\delta n$ is the nonlinear deformation of the concentration profile, $ReE_l$ is the profile of the longitudinal electric field (real part), $z_R$ is the reflection point, $z_0$ is the resonant point, $\theta$ is the angle of incidence of the signal electromagnetic wave, RS and BS are the Stokes waves corresponding to RS and BS processes, respectively.

In the present paper we develop this idea further. We suggest a bitlike data transfer scheme, where the incident signal consists of equally spaced narrow pulses. The message coding on the sequence of pulsed wave trains is similar to the soliton-based scheme of information transmission about to be used for long-distance fiber optics communications. The advantages of the proposed scheme are as follows:

1. The peak intensity of the short pulses (both signal and probe beams) can be much greater than for the continuous wave radiation. Since the Raman signal intensity is proportional to the product of the incident and probe beam intensities, it is then easier to detect.

2. It is possible to induce pulse compression of the incident signal train during its propagation by using an appropriate initial frequency chirp. As a result, there will be an increased signal intensity and therefore scattered Raman signal intensity.

3. The generation of the intense Langmuir wave near the point of resonant absorption creates a pulse of ponderomotive force pushing the plasma out of the resonance region. This will produce a sound wave pulse propagating through the sheath, which may also be detected on the vehicle.

4. In addition, the distortion of the plasma profile will scatter the probe wave. This process is known as Brillouin scattering (BS). On-board detection of the BS signal can serve as an independent channel of information transfer.

5. The strong electric field in the resonant region can produce fast electrons via wave breaking. The energy of these fast electrons is well above the thermal energy, and the collision cross section is small. Therefore the fast electrons can penetrate the plasma slab and be detected at the vehicle.

6. Bitlike data transfer is less sensitive to the noise. The noise/signal requirements are much weaker when several independent information channels are available.

The paper is organized as follows. In Section 2 we describe the generation of the Langmuir wave in the process of reflection and resonant absorption of the incident pulse of the electromagnetic wave sent by a powerful ground transmitter. Section 3 is devoted to the study of radiative nonlinear deformation of the plasma density profile by the radiative forcing of Langmuir waves. In Section 4 we consider the RS of the pump wave on the Langmuir oscillations and the BS of the probe wave from the deformation of the plasma density profile. The probe (pump) wave originates on the vehicle. We study the dependence of RS and BS on pulse duration and derive the optimal value for this parameter. In Section 5 we consider the generation and propagation of acoustic pulses accompanying the process of nonstationary deformations of the plasma density profile. In Section 6 we study the breaking of Langmuir waves and the generation of fast electrons by the wave breaking. In Section 7 we discuss how the signal wave could be compressed by the proper pulse chirp to achieve higher intensities of this wave in the plasma sheath, thereby amplifying the nonlinear interactions needed for scattering. In Section 8 we summarize our re-

sults and make estimates of the amplitudes of the different nonlinear processes and study their relative efficiencies for the purposes of communication through plasma sheaths.

## Linear Transformation of the Short Pulse

An electromagnetic wave propagating in an inhomogeneous plasma can be converted into a Langmuir wave. This process is often referred to as resonant absorption (RA). The usual pattern of the linear transformation can be summarized as follows, [e.g., *Ginzburg*, 1964; *Kruer*, 1988, *Hinkel-Lipsker et al.*, 1992]. A p-polarized electromagnetic wave incident on the plasma at an angle $\theta$ reaches a reflection point where $n = n_c \cos\theta$, $\omega_p(n_c) = \omega_0$. The wave tunnels to the conversion point, $n = n_c$, where the excitation of the electron density oscillations $\delta n$ with frequency $\omega = \omega_p$ occurs. The plasma wave amplitude can be found from the equations

$$\mathrm{div} D = \mathrm{div}(\epsilon(\omega)E) \doteq 0, \quad (1)$$

$$\mathrm{div} E = -4\pi e \delta n, \quad (2)$$

$$\delta n = \frac{E \,\mathrm{grad}\epsilon}{-4\pi e \epsilon(\omega)}, \quad (3)$$

where $\epsilon(\omega)$ is the dielectric response function. We see that at the resonant point, $\epsilon \to 0$, $\delta n \to \infty$ provided that the electric field has a component along the density gradient. From Maxwell's equations, one can see [*Ginzburg*, 1964] that in the case of a linear density profile,

$$n(z) = n_0(1 - z/L), \quad (4)$$

and we have $E_z \propto 1/z$, $E_y \propto \ln z$ and $H = H_0$ in the vicinity of the resonance. Here we measure the coordinate $z$ from the resonant point, which implies

$$\omega_0 = \omega_p(0) = (4\pi e^2 n_0/m)^{1/2}, \quad (5)$$

where $\omega_0$ is the frequency of the signal electromagnetic wave, $\omega_p$ is the plasma frequency, and $e$ and $m$ are the elementary charge and the electron mass, respectively.

To this point, we have not considered any mechanisms limiting the plasma wave growth driven by the electromagnetic pump. Within the linear theory, the plasma wave amplitude is limited by collisional damping or by convection from the resonant zone. In cold plasmas the first mechanism dominates when the collisional frequency is large enough so that $\nu/\omega \gg (r_d/L)^{2/3}$, where $r_d = v_{Te}/\omega_p$ is the Debye radius. The collisional frequency includes both the electron-ion and electron-neutral collisions. Without thermal dispersion, $\epsilon$ is given by

$$\epsilon = 1 - \frac{\omega_p^2}{\omega(\omega + i\nu)}. \quad (6)$$

From (2), (3), and (6), one can estimate the resonance width $\Delta z$ and the maximal amplitude of the plasma wave $E_z$ max as follows

$$\Delta z \sim L\nu/\omega_0, \quad (7)$$

$$E_z \max \sim E_0 \omega_0/\nu.$$

where $E_0$ is the electric field of the incident electromagnetic wave. In the vicinity of the maximum, the solution for the longitudinal electric field is given by the following expression [*Ginzburg*, 1964; *Kruer*, 1988, *Hinkel-Lipsker et al.*, 1992]:

$$E_z = \frac{i \sin\theta \, E_0 \Lambda \exp(-i\omega_0 t)}{z/L - 1 + i\nu/\omega_0}, \quad (8)$$

where $\nu$ is the electron collision frequency and $\theta$ is the angle of incidence of the signal electromagnetic wave. The value $\Lambda$ is a magnetic field factor:

$$\Lambda = -Ai'(q)/[1 - iq\pi^2 Ai'(q)(Bi'(-q) + iAi'(-q))], \quad (9)$$

where

$$q = (k_0 L)^{2/3} \sin^2\theta;$$

$$k_0 = \omega_0/c$$

and the prime denotes differentiation with respect to the argument. Here $Ai$ and $Bi$ are Airy functions. The absolute value of $\Lambda$ measures the fraction of the signal wave electric field that pen-

etrates to the resonant point $z = 0$. The function $\Lambda(q)$ is tabulated, see *[e.g., Kruer, 1988; Hinkel-Lipsker et al., 1992]*, and a reasonably good approximation is $\Lambda(q) = 2.3\sqrt{q}\exp(-\frac{2}{3}q^2)$. The function $\Lambda(q)$ has a maximum at $q_{\max} = 0.64$ with the width $\Delta q \sim q_{\max}$, so that

$$\sin^2\theta \approx 0.64(k_0L)^{-2/3}, \quad |\Lambda| \approx 1.2. \qquad (10)$$

In the situation where convection of Langmuir waves dominates over the dissipation, one may still use (7)-(9), with $\nu$ replaced by $\nu_{\text{eff}} = \omega_p(r_d/L)^{2/3}$ *[Ginzburg, 1964, Kruer, 1988; Hinkel-Lipsker et al., 1992]*. whereas, in what follows, we will consider only the case $\nu > \nu_{\text{eff}}$, unless otherwise noted, our results are valid also when the opposite inequality holds (with $\nu$ replaced by $\nu_{\text{eff}}$).

We now consider the case where the signal electromagnetic wave has a form of the Gaussian pulse,

$$E_{\text{signal}} = E_0\exp(-i\omega_0 t) = A_0 \exp(-t^2/2\tau^2 - i\omega_0 t). \qquad (11)$$

* Since our problem is linear, we can use (9) for each Fourier harmonics of the pulse (12) and then integrate over all the harmonics according to the superposition principle. Note, however, that different harmonics will have different resonant points, and this will result in a widening of the resonance with corresponding reduction in the peak amplitude. Taking into account that the Fourier transform of (12) is

$$E_\omega = A_0\tau\exp(-(\omega-\omega_0)^2\tau^2/2), \qquad (12)$$

and proceeding with the method we have described, we arrive at the equation for the electric field of the Langmuir oscillations:

$$E_z = \int E_{l\omega}d\omega = iA_0\tau\sin\theta \int \frac{\Lambda_\omega\exp(-i\omega t - (\omega-\omega_0)^2\tau^2/2)}{z/L - 1 + \omega_0^2/\omega^2 + i\nu/\omega}\,d\omega. \qquad (13)$$

Suppose that both $\nu$ and $\tau^{-1}$ are small compared to $\omega_0$. Then, the integrand of (14) will have a highly pronounced resonance on $\omega = \omega_0$, so that we can disregard the frequency dependence of the factor $\Lambda$ (which is a smooth function of $\omega$) and simply take it at the resonant point. By straightforward algebraic manipulations we can reduce (14) to the form

$$E_z = \frac{i\pi^{1/2}}{2}\omega_0\tau A_0\sin\theta\,\Lambda(\omega_0) \exp(-t^2/2\tau^2 - i\omega_0 t)Z(\eta), \qquad (14)$$

where

$$Z(x) = \pi^{-1/2}\int_{-\infty}^{+\infty}\frac{\exp(-\lambda^2)\,d\lambda}{(\lambda - x)}, \qquad (15)$$

and

$$\eta = -\frac{z\omega_0\tau}{2\sqrt{2}L} - \frac{it}{\sqrt{2}\tau} + \frac{i\nu\tau}{2\sqrt{2}}. \qquad (16)$$

As is usual for initial value problems, the integral (16) must be taken along a contour below the pole of the integrand. The exponential factor in (15) mimics the time slope of the signal pulse. However, the original shape is distorted in $E_l$ due to the time dependence of $Z(\eta)$ through $\eta$. Remarkably, the function $Z(\eta)$ coincides with the plasma dispersion function which appears in the theory of plasma stability *[Fried and Conte, 1961]*. For a qualitative analysis and many practical applications, one can use the so-called two-pole approximation for this function, an approximation which is often exploited in plasma stability theory, namely

$$Z(\nu) = \frac{0.5 + 0.81i}{a - \nu} - \frac{0.5 - 0.81i}{a^* + \nu}, \qquad (17)$$

where

$$a = 0.51 - 0.81i. \qquad (18)$$

Let us analyze the limit $\nu\tau \gg 1$. Using the asymptotic expansion of $Z$ for large arguments, $Z(\eta) \approx -1/\eta$ *[Fried and Conte, 1961]*, we arrive at

$$E_z \approx A_0\frac{i\sin\theta\,\Lambda\exp(-t^2/2\tau^2 - i\omega_0 t)}{z/L - 1 + i\nu/\omega_0}. \qquad (19)$$

Observe that (20) could be obtained by substitution of (12) into (9). Therefore the long pulses

may be treated as quasi-monochromatic, and one can use the results of the previous papers [*Ginzburg*, 1964, *Kruer*, 1988, *Hinkel-Lipsker et al.*, 1992; *Fried and Conte*, 1961]. The maximum of electric field is located at $z = 0$, and is given by (8). The width of the electric field peak is given by (7).

At the resonant point, $z = 0$, the argument $\eta$ is small and, therefore, we use the following expression for the $Z$ function:

$$Z(\eta) \approx i\pi^{1/2} \exp(-\eta^2) - 2\eta \qquad (20)$$

According to this expression, in the case where $\nu\tau \ll 1$ we have $Z(\eta) \approx i\pi^{1/2}$, so that the Langmuir electric field tends to a value which is independent of $\nu$ and is $\omega_0\tau$ times greater than the amplitude of the incident wave $E_0$. This result is readily understood from the physical point of view. For short (compared to the mean time between collisions) pulses, $\nu\tau \ll 1$, the spectral width $\delta\omega \sim 1/\tau$ exceeds the width of the resonant zone, $\Delta\omega \sim \nu$, and only a small fraction, $\nu/\delta\omega \sim \nu\tau$, of the pulse field drives the Langmuir wave. Therefore $E_0$ in (8) must be replaced by $E_0\nu\tau$, so that

$$E_z \ \max \sim E_0\omega_0\tau. \qquad (21)$$

For the same reason, the width of the electric field peak is now broader than in the case $\nu\tau \gg 1$,

$$\Delta z \sim L/\omega_0\tau. \qquad (22)$$

In Figure 2 we present spatial profiles of $E_z$ for several different values of $\nu\tau$ at $t = 0$. One can see that the maximum of the field is located at $z = 0$ and the profile is symmetric. In accordance with (7) and (23), pulse width $\delta z$ decreases when $\tau$ is increasing and saturates at an independent of $\tau$ value when $\nu\tau \gg 1$. It follows also from (7) and (23) that for fixed $\tau$, $\delta z$ increases linearly with $\nu$ when $\nu\tau \gg 1$ and saturates when $\nu\tau \ll 1$. In Figure 3 we show the evolution of $E_z$ at the resonant point, $z = 0$, for several pulses of different duration, as calculated from (14).

It is remarkable that the total energy absorption in the process of the linear transformation,

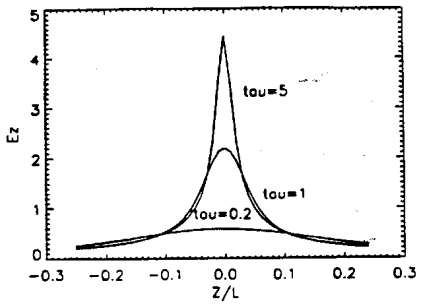


**Figure 2.** The spatial profiles of the longitudinal electric field $E_z$ for several different values of the pulse duration $\tau$ at time $t = 0$. The carrier wave frequency and the frequency of collisions are $\omega_0 = 100$ and $\nu = 1$ respectively.

$$Q = \nu \int \frac{|E|^2}{8\pi}\, dz\, dt, \qquad (23)$$

is independent of $\nu$, similar to the case of the monochromatic wave. Indeed, substituting (15)-(17) in (24) and integrating, we obtain

$$Q = \frac{1}{8}\sqrt{\pi} A_0^2 \omega_0 \tau \Lambda_{\omega_0}^2 \sin^2\theta. \qquad (24)$$

This result is natural because the contribution of individual harmonics is $\nu$ independent, and so is

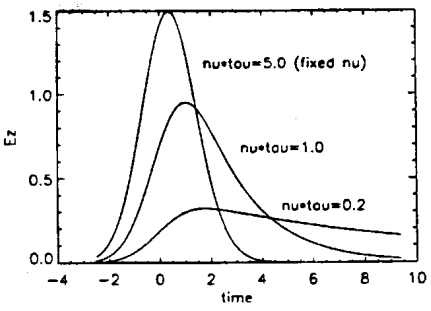


**Figure 3.** The evolution of the longitudinal electric field $E_z$ at the resonant point $z = 0$ for several pulses of different duration and collision frequency $\nu = 1$.

the integral over all the harmonics contributing to $Q$.

## Radiative Deformation of the Electron Density Profile

The most important nonlinear effect neglected in the linear approximation is the plasma profile deformation under the action of ponderomotive forcing with equivalent pressure,

$$P_{\text{rad}} = \frac{|E_z|^2}{16\pi}. \tag{25}$$

This effect essentially modifies the linear mechanisms of the electromagnetic wave conversion into the Langmuir oscillations when the profile deformation, $\delta n$, becomes comparable with the density variation, $\Delta n$, on the scale of the resonant zone, $\Delta z$, or

$$\delta n/n_0 \sim \Delta z/L. \tag{26}$$

The force (26) acts on the ion component of the plasma via an ambipolar electric field (preserving the quasi-neutrality of the plasma). Combining the motion equation for ions with the ion continuity equation we obtain the following equation for the evolution of $\delta n$ [*Zakharov*, 1984]:

$$\partial_t^2 \delta n - C_s^2 \partial_z^2 \delta n = \frac{1}{16\pi M} \partial_z^2 |E_z|^2, \tag{27}$$

where

$$C_s = \sqrt{T_e/M} \tag{28}$$

is the speed of the ion sound, $M$ is the ion mass and $T_e$ is the electron temperature.

Let us consider first the case of long (compared to the time between the electron-neutral collisions) pulses, $\nu\tau \gg 1$. Then the maximum electric field of Langmuir oscillations and the width of the resonance are given by (8) and (7). When the pulse duration $\tau$ is larger than the time of the sound passage through the resonance zone, $\tau > \Delta z/C_s = L\nu/C_s\omega_0$, the static approximation, $\partial_t \approx 0$, to (28) is applicable, and we obtain

$$\delta n \approx -\frac{|E_z|^2}{16\pi M C_s^2}. \tag{29}$$

Substituting (8) into (30) we have

$$|\delta n|\max \sim \frac{E_0^2}{16\pi M C_s^2} \left(\frac{\omega_0}{\nu}\right)^2. \tag{30}$$

From (27), (7), and (31) we see that for very moderate field strength,

$$\frac{E_0^2}{16\pi n_0 T_e} > \left(\frac{\nu}{\omega_0}\right)^3, \tag{31}$$

the linear approximation breaks [*Sagdeev*, 1991].

In the opposite to the static approximation limit, $\tau \ll L\nu/C_s\omega_0$, the value of $\delta n$ is determined by a balance of the ion inertia (the first term in (28)) and the ponderomotive force, and we get

$$|\delta n|\max \sim \frac{E_z^2}{16\pi M} \frac{\tau^2}{(\Delta z)^2}. \tag{32}$$

Using (27), (7), and (33), we see that now the critical amplitude is inversely proportional to the second power of the pulse duration $\tau$,

$$\frac{E_0^2}{16\pi n_0 M} > \frac{\nu^5 L^2}{\tau^2 \omega_0^5}. \tag{33}$$

For short (compared to the collision time) pulses, $\nu\tau \ll 1$, the maximum field and the resonance width are given by (22) and (23). When the static approximation is applicable, $\tau > \Delta z/C_s = L/C_s(\omega_0\tau)$, we have the following expression for the critical field from (27), (23), and (30):

$$\frac{E_0^2}{16\pi n_0 T_e} \sim \frac{1}{(\omega_0\tau)^3}. \tag{34}$$

If $\tau < \Delta z/C_s = L/C_s(\omega_0\tau)$, then $\delta n$ is determined by the ion inertia. From (27), (23), and (33) we see that the critical field, for which the linear approximation breaks, very rapidly increases with the decrease of the pulse duration $\tau$,

$$\frac{E_0^2}{16\pi n_0 M} \sim \frac{I^2}{\omega_0^5 \tau^7}. \tag{35}$$

After pulse termination a density well on the plasma profile persists. Such a density perturbation will decay into two sonic waves propagat-

ing in the opposite directions. This process will be considered in Section 5.

If the linear approximation breaks, the saturation level for $\delta n$ is determined by quite complicated nonlinear dynamics. Recent numerical and analytical studies and the previous works on the nonlinear resonant absorption are reviewed by *Gavrilov et al.* [1995]. Usually, the maximum of $\delta n$ in the nonlinear regime is reached in the long-pulse regime, $\tau \gg L\nu/C_s\omega_0$, and, as in the linear regime, is determined by the ballance of the ponderomotive and gasodynamical pressures (see (30)). In this case, a sharp jumplike distortion is formed on the plasma profile, so that the reflection point and the resonant point approach very closely to each other. On the other hand, 1-dimensional (1-D) density jumps are unstable with respect to the two-dimensional (2-D) (and three-dimensional (3D)) perturbations. The instability can result in a collapse-type intensification of the disturbances, so that in reality $\delta n$ may happen to be much greater than the one determined by (31) and (33).

If the inverse frequency of the ion-neutral collisions becomes smaller than the pulse duration $\tau$, then the ion-neutral collisions will glue together the ion and neutral component. In such a case we need to replace the ion-sound speed $C_s$ by the regular speed of sound in gases, $C_n = \sqrt{T_n/M_n}$, in (28), and $\delta n$ has to be understood now as a sum of the concentration disturbances corresponding to the ion and neutral components, $\delta n = \delta n_i + \delta n_n$. This means that there will be larger (by the factor of $C_s^2/C_n^2$ or $T_e/T_n$ if $M_n = M$) disturbances $\delta n$, and smaller (by the factor of $n_i C_s^2/nC_n^2$) disturbances $\delta n_i$.

## Raman and Brillouin Scattering

Density disturbances $\tilde{n}$ related with either Langmuir oscillations or/and the profile modifications discussed in the previous section will scatter the high-frequency electromagnetic pump wave sent from the vehicle. These processes are called the Raman scattering (RS) and Brilluoin scattering (BS) respectively. The electric field of the scattered wave, $E_s$, may be evaluated on the basis of the following equation

$$\partial_{tt}^2 E_s - c^2 \partial_{zz}^2 \partial E_s = -\frac{\omega_p^2}{n}\tilde{n}E_{pm}, \qquad (36)$$

where $E_{pm}$ is the field of the electromagnetic pump wave sent from the vehicle,

$$E_{pm} = A_{pm}\exp(ik_{pm}(z - ct)) + c.c., \qquad (37)$$

and $k_{pm}$ is the wavenumber of the pump wave. For simplicity, we assume $ck_{pm} \gg \omega_p$. For small intensities of the scattered signal one can neglect its feedback on the amplitude of the pump wave. Then (37) becomes linear and can be solved analytically. Another simplification comes from the fact that the function $\tilde{n}(z)$ has a sharp peak, which is thinner than the wavelength $2\pi/k_{pm}$, and a condition, $\omega_0\tau \gg 1$. Then, one can neglect the time-derivative term compared to the second term in (37) in the region of the scattering. Therefore one can use the following equation

$$c^2\partial_{zz}^2 E_s = \frac{\omega_p^2}{n}\tilde{n}E_{pm}, \qquad (38)$$

where $E_{pm}$ can also be considered independent of $z$ in the excitation zone. Note that the regime under consideration is very different from the one usually considered in the theory of laser-plasma interactions [*Kruer*, 1988]. In the latter case, it is accepted that the distributions are smooth on the wavelength of the electromagnetic wave so that the envelope approximation can be used.

## Raman Scattering

Let us consider first the RS process. In this case the density perturbation $\tilde{n}$ is caused by the Langmuir oscillations and related to the field via the Gauss law,

$$\tilde{n} = -\frac{1}{4\pi e}\partial_z E_z. \qquad (39)$$

Substituting this expression into (39) and integrating twice we get

$$E_s = \frac{eE_{pm}}{mc^2}\int_{-\infty}^{+\infty} E_z\, dz. \qquad (40)$$

From this equation we obtain the following estimate for the scattered signal,

$$E_s' \sim \frac{eE_{pm}}{mc^2} E_Z \ \max\Delta z. \tag{41}$$

Note that, according to (8), (7), (22), and (23), the product $E_Z \ \max\Delta z$ is not sensitive to the value of $\nu\tau$. Substituting (8) and (7) (or (22) and (23)) in (42), we arrive at the following expression for the RS signal:

$$E_s \sim \frac{eE_{pm}}{mc^2} E_0 L. \tag{42}$$

This expression coincides with the formula for the Raman signal obtained for the infinitely long pulse by *Nazarenko et al.* [1994].

### Brillouin Scattering

Let us evaluate now the BS signal resulting from the scattering of the pump wave by the nonlinear deformations of the plasma profile considered in section 3. If the width of the resonant zone is smaller than the wavelength of the pump wave, $k_{pm}\Delta z \ll 1$, one can estimate the amplitude of the scattering signal, $E_s$, using (39):

$$E_s \sim \frac{\omega_p^2}{c^2 n} (\Delta z)^2 \tilde{n} E_{pm}. \tag{43}$$

According to section 3, the values of $\tilde{n}$ and $\Delta z$ are different for different pulse durations $\tau$. Consider first the case $\nu\tau > 1$, $\tau > L\nu/C_s\omega_0$. Substituting (7) and (31) in (44) we get

$$E_s \sim \frac{\omega_p^2 L^2}{16\pi n c^2 T_e} E_0^2 E_{pm}. \tag{44}$$

In the case $\nu\tau > 1$, $\tau < L\nu/C_s\omega_0$ we have from (8), (7), (33), and (44)

$$E_s \sim \frac{\omega_p^2 \omega_0^2 \tau^2}{16\pi\nu^2 n c^2 M} E_0^2 E_{pm}. \tag{45}$$

For $\nu\tau < 1$, $\tau > L/C_s(\omega_0\tau)$, we must use (22), (23), and (30) in the estimate (44), and we obtain

$$E_s \sim \frac{\omega_p^2 L^2}{16\pi n c^2 T_e} E_0^2 E_{pm}. \tag{46}$$

Finally, in the case $\nu\tau < 1$, $\tau < L/C_s(\omega_0\tau)$, we use (22), (23), and (33) to get

$$E_s \sim \frac{\omega_p^2 \tau^4 \omega_0^2}{16\pi n c^2 M} E_0^2 E_{pm}. \tag{47}$$

As we see, BS is most effective if the pulse duration is greater than time of the sound passage through the resonant zone. In this case, according to (45) and (47), the BS signal is not sensitive to the parameter $\nu\tau$. According to (46) and (48), the BS signal decreases with decreasing $\tau$ if the pulse duration is smaller than the time of the sound passage through the resonant zone.

### Generation of Ion-Acoustic Waves

As discussed in section 3, the ponderomotive pressure creates a density well near the resonant point. If $C_s\tau \gg \Delta z$, the density variation adiabatically follows the profile of the ponderomotive pressure. This means that after the pulse termination, the density depression will also terminate. The situation is different for shorter pulses. In that case, the profile of the density depression will be determined by the ion inertia rather than the pressure balance. Such a density perturbation will remain after the pulse termination. Its amplitude is given by formula (33).

After the pulse termination, the density depression will decay into perturbations propagating in all directions away from the resonant zone in the form of either an ion-acoustic wave or neutral sound, if the rate of the ion-neutral collisions is high (see the end of section 3). The part of the acoustic wave propagating toward the vehicle can be detected, in principle, on board the vehicle using a microphone. This serves as an independent channel of information transfer. The optimal pulse duration for this channel is defined by the condition that the density depression is large, but at the same time the pressure is not balanced and the inertia is important. Such an optimal pulse duration is given by the condition

$$C_s\tau_{\text{opt}} = \Delta z. \tag{48}$$

The same or greater amount of time is required

for the time separation of pulses due to the finite time of the profile relaxation to its initial (unperturbed) shape. Of course, the acoustic signal has to compete with the background acoustic noise produced by turbulence. Nevertheless, the knowledge of the interval between the pulses and other characteristics of the incoming signal wave may strongly improve sensitivity of this communication channel through the analysis of correlations.

The propagation of the density disturbance after the pulse termination can be calculated using the density depression, produced by the ponderomotive forcing, as an initial condition for the wave equation (28) (without the right-hand side). In the simplest one dimensional case with constant speed of sound the solution is given by the D'Alambert formula:

$$\delta n(z,t) = \frac{1}{2}\delta n(z + C_s t, 0) + \frac{1}{2}\delta n(z - C_s t, 0). \tag{49}$$

Here, $\delta n(z,0)$ can be calculated from the well-known solution of the 1-D wave equation with a localized time-dependent source for $t \leq 0$. Solving the problem in the 2-D and 3-D cases is also quite straightforward if $C_s$ = const. On the other hand, the inhomogeneity of the plasma temperature can lead to the refraction and reflection of the acoustic rays due to the variations in $C_s$ [*Landau and Lifshitz*, 1987]. Another mechanism which can cause refraction and reflection is related to the convective distortion of the wave fronts by the sheared velocity field surrounding the vehicle [*Landau and Lifshitz*, 1987]. This mechanism appears to be very important because of the supersonic nature of the flow. Note that to estimate the influence of the sheared flow one should evaluate the effective Mach number, $M_{eff}$, by the speed of sound inside the sheath. Such a Mach number may appear to be up to one order of magnitude smaller than the Mach number calculated by the sound speed in the atmosphere, that is, 330 m/sec. The influence of the sheared velocity field is large if $M_{eff}$ is of order one or greater. The effect of the temperature gradient and the shear flow need further careful investigation.

If the temperatures of the electrons and ions in plasma are nearly the same (isothermal plasma) then the ion-sonic wave may be heavily damped. Strictly speaking, the initially strong linear Landau damping may be followed by a weakly decaying state corresponding to plateau formation on the particle distribution function. Therefore, to make a conclusion about the dissipation rate of the ion sound, one has to compare the characteristic time of the plateau formation and the smaller of the pulse duration and the time of plateau decay (inverse collision frequency). The characteristic time of the plateau formation can be calculated from the quasilinear theory if the spectrum of the ion sound is broad or from the theory of nonlinear Landau damping for a nearly monochromatic acoustic wave. If the characteristic time of plateau formation is large compared to the inverse collision frequency then the ion-acoustic wave will quickly decay transferring its momentum to the neutral component. This momentum perturbation will propagate on the neutral component in the form of a neutral sonic wave (as in the case of large ion-neutral collisions discussed above). Due to momentum conservation, the velocity perturbations associated with this wave will be $n_n/n_i$ times smaller than in the ion-sonic wave ($n_i$ and $n_n$ are the densities of ion and the neutral components respectively). If the characteristic time of plateau formation is smaller than the inverse collision frequency then the ion-sonic wave will be weakly damped and can be detected on board. The ion-sonic wave can be detected by electric field measurements as well as with a microphone.

## Wave Breaking and Generation of Fast Electrons

Because of resonant amplification, the local longitudinal electric field in plasmas can be so large that the electron quiver velocity may easily exceed the thermal velocity. Then wave breaking can play a significant role, limiting the electric field amplitude and producing fast electrons. The fast electrons can be detected on the vehi-

cle because their collision cross section is small and they easily penetrate the plasma slab. The Earth's magnetic field also cannot trap the fast electrons, since the their Larmor radius is at least 1 order of magnitude greater that the thickness of the plasma slab.

Wave-breaking can be examined using the equation for the Lagrangian displacement of the fluid element $\delta(x_0, t)$ under the action of the external field [*Albriton and Koch*, 1975]:

$$\partial_t^2\delta + \omega_p^2(z_0)\delta = \frac{eH_0}{m}\sin\theta\sin\omega_0 t, \qquad (50)$$

where $H_0$ is the amplitude of the magnetic field field oscillations in the incident wave, and $\theta$ is the angle of incidence. Since the wave breaking process takes many plasma wave periods, we can introduce the envelope $a$ of the displacement and assume it is slowly varying;

$$\delta = a\exp(-i\omega_0 t) + c.c., \quad \partial_t a \ll \omega_0 a. \qquad (51)$$

Then, in the lowest order, (51) can be rewritten as

$$\partial_t a + \nu a - i\omega_0\frac{\Delta\omega_p}{\omega_0}a = -\frac{i\omega_0}{4}r_0(t). \qquad (52)$$

Here

$$r_0 = \frac{eH_0\sin\theta}{m\omega_0^2}, \quad \frac{\Delta\omega_p}{\omega_0} = \frac{\omega_p}{\omega_0} - 1 \approx \frac{z_0}{2L}. \qquad (53)$$

We introduced also a damping of the plasma waves. For a monochromatic wave, the displacement $\delta$ is related with the local field by $\delta = eE/m\omega_0^2$. Substitution of this expression into (51) gives the familiar expression for the longitudinal electric field (9). On the other hand, the actual electric field may saturate at a lower level due to the wave breaking process. Integrating (53), we have

$$a = -\frac{i\omega_0}{4}\exp(i\Delta\omega_p t) \qquad (54)$$
$$\int_0^t r_0(\tau)\exp(i\Delta\omega_p\tau - \nu(t-\tau))\,d\tau,$$

or

$$\delta = -\frac{i\omega_0}{4}\exp(-i\omega_0 t)\int_0^t r_0(\tau) \qquad (55)$$
$$\exp((i\Delta\omega_p - \nu)(t-\tau))\,d\tau.$$

Wave breaking time and location are defined by conditions [*Landau and Lifshitz*, 1987]

$$\partial_{z_0}\delta = -1, \quad \partial_{z_0^2}^2\delta = 0. \qquad (56)$$

It is natural that the wave breaking takes place at $z_0 \approx 0$. For the time of the wave breaking we have an equation

$$\frac{\omega_0^2}{4L}\int_0^t r_0(\tau)(t-\tau)\exp(-\nu(t-\tau))\,d\tau = 1. \qquad (57)$$

In the case of the constant field, condition (58) gives

$$1 = \frac{\omega_0^2 r_0}{4L}\int_0^t (t-\tau)\exp(-\nu(t-\tau)) \qquad (58)$$
$$d\tau = \frac{\omega_0^2}{\nu^2}\frac{r_0}{4L}[1 - \exp(-\nu t)(1+\nu t)].$$

When $\nu \to 0$, the expression (59) reproduces the well-known result $\tau_0$, $\omega_0\tau_0 = (8L/r_0)^{1/2}$ for the wave breaking time [*Albriton and Koch*, 1975] . For finite $\nu$, (59) has a solution if

$$\frac{\omega_0^2}{\nu^2}\frac{r_0}{4L} > 1. \qquad (59)$$

This means that the damping can stop the wave-breaking process if $\tau_0$ is comparable or greater than the plasmon lifetime, $\nu\tau_0 > 1$.

The electron velocity, accelerated by the wave breaking, is

$$\frac{d\delta}{dt} \sim v_d(2L/r_0)^{1/2}, \qquad (60)$$

where $v_d = eH_0\sin\theta/m\omega_0$. The velocity (61) is very large, and such fast electrons can be easily detected in experiments [*Lee et al.*, 1986]. Note that the primary breaking described above generates fast electrons moving down the plasma density gradient. Nevertheless, secondary wave breaking, immediately following the first one,

creates fast electrons moving in all directions, so that they can be detected on board the vehicle.

## Compression of a Chirped Pulse

It is a well-known fact in nonlinear optics that a pulse across which the frequency varies, called a chirped pulse, is compressed during its propagation in a dispersive medium. Recently, *S. C. Cowley et al.* (preprint,1993) have suggested to use this effect for compression and, therefore, amplification of the pulse as it propagates toward the critical density. The mechanism of compression is very simple. Let us consider a pulse with its frequency minimum in the center. The front of the pulse will have higher group velocity, and will be reflected deeper in the plasma. It can be arranged that after reflection, the front will arrive at the reflection point of the pulse center at the same time as the center of the pulse. Further, the rear of the pulse will also have velocity higher than the central part, and can catch up the pulse center at the moment of its reflection. Therefore, all the parts of the pulse will focus at this moment.

The dispersion of the electromagnetic wave is maximal in the vicinity of the reflection point. Considering the linear density profile, (4), *S. C. Cowley et al.* (preprint, 1993) found that the amplification factor for the pulse intensity is

$$g \approx (k_0 L)^{2/3}. \tag{61}$$

We see that the considerable gain can be achieved if $k_0 L > 1$. On other hand, one should not use excessively large values of $k_0 L$ because then the process of conversion into the plasma waves becomes very sensitive to the angle of incidence.

## Summary and Estimates

It has been shown above that the resonant amplification of the electric field in the vicinity of the transformation point can be used for information transfer through a plasma sheath in a variety of ways. Each of the communication channels exploits in some way nonlinear effects. Therefore the amplitude of the electromagnetic wave must be large enough to provide detectable levels of the signal. Estimates of the signal level have to be done for available powers of electromagnetic wave generators to understand how realistic the proposed methods are.

First, let us discuss the optimal wavelength of the incident electromagnetic wave. We argue that the optimal choice is $k_0 L \sim 1$. In this case the penetration (tunneling) to the transformation zone will be insensitive to the angle of the wave incidence, determined by the mutual orientation of the vehicle and the transmitter. In the case $k_0 L \gg 1$ the angle of incidence must be precisely tuned, otherwise the field penetrating to the transformation point will be exponentially small. Also, much more powerful generators are available at the lower frequencies, which favor the choice of $k_0 L \sim 1$. Finally, high-frequency radiation is severely damped in the atmosphere because of the resonant excitation of the molecular bonds of water vapor.

Therefore for a typical scale of the plasma profile $L \sim 10$ cm [*e.g., Papa and Taylor*, 1974], the wavelength $\sim 10$ cm has to be used. For such wavelength values, pulse generators with peak power $P \sim 100$ MW are readily available. These generators can be mounted on a jumbo jet. The wave intensity at the vehicle location will be determined by the diffraction size $d$, given by

$$d = (\lambda D)^{1/2}, \tag{62}$$

where $D$ is the distance between the vehicle and the transmitter antenna. With the initial antenna size $d_0$, the power density in the vehicle vicinity drops by a factor of $\lambda D/d_0^2$. For a distance $D \sim 100$ km and transmitter power $P \sim 100$ MW the wave intensity in the vehicle vicinity will be

$$I \sim \left(\frac{P}{10^8 W}\right) \frac{1}{(\lambda/10\text{cm})(D/100\text{km})} \text{W/cm}^2. \tag{63}$$

Therefore an intensity of approximately 1 W/cm$^2$ can be achieved.

To estimate the amplitudes of the RS and BS signal we express the electric field of the electromagnetic wave in terms of its power according to

$$I = \frac{1}{2}\epsilon_0 c E_0^2, \qquad \text{SI units}, \tag{64}$$

or, substituting numbers for $\epsilon_0$ and $c$,

$$E_0 = 2745\sqrt{I[\mathrm{W/cm^2}]}\mathrm{V/m}. \tag{65}$$

Substituting (66) and $mc^2 = 0.5$ MeV into (43) and assuming $k_{pm}L \sim 1$, we get for the relative amplitude of the RS signal

$$E_{RS}/E_{pm} \sim 0.5 \times 10^{-2} \tag{66}$$

From (43) and (45) we can estimate the relative efficiency of the BS and RS in the long (longer than all the characteristic times in the problem) pulse regime,

$$E_{\mathrm{BS}}/E_{\mathrm{RS}} \sim \frac{eLE_0}{4T}. \tag{67}$$

Substituting $T \sim 1$ eV and $E_0 \sim 2700$ V/m we find that the BS signal is 3 orders of magnitude stronger than the RS signal. Formal substitution of the above parameters into (45) results in a BS electric field which is stronger than the field of the pump wave. This fact simply indicates that we went beyond the limits of applicability of (45), which uses the assumption of the smallness of $E_s$ compared to $E_{pm}$. In this case, the scattered signal is of the same order of magnitude as the pump wave, which is very strong.

As we see, the power of the BS signal is most likely to appear much greater that the power of the RS signal. On the other hand, the perturbations responsible for the BS signal (that is the profile disturbances due to the ponderomotive forces) have low frequency and have to compete with strong acoustic turbulent noise present in the plasma flow surrounding the vehicle. The perturbations responsible for the RS signal (that is, Langmuir oscillations) oscillate at the plasma frequency which is much higher than the frequency of acoustic turbulence. Therefore the RS signal can be efficiently extracted by selective receipt of the scattered signal at the frequency $\omega_{pm}+\omega_p$ or $\omega_{pm}-\omega_p$. Note that the noise at the frequency $\omega_p$, that is the Langmuir turbulence, remains at the low-level $E_{th}$ of thermal fluctuations, which can be found from the following relation

$$\frac{\epsilon_0 E^2}{2} = \frac{nk_BT}{N_D}, \qquad \text{SI units}, \tag{68}$$

where $k_B$ is the Boltzmann's constant and $N_D$ is the number of particle in a Debye sphere,

$$N_D = 1.72 \times 10^9 T^{3/2}[\mathrm{eV}]n^{-1/2}[\mathrm{cm^{-3}}]. \tag{69}$$

Substituting $n \sim 10^{11}$ cm$^{-3}$ and $T = 1$ eV in (70), we get $N_D \sim 5 \times 10^3$. Then, from (69) we have the following value for the thermal level of Langmuir oscillations,

$$E_{th} \sim 500\mathrm{V/m}. \tag{70}$$

Assuming $E_0 \sim 2700$ V/m and $\omega_0/\nu = 100$, we have the following value for the field of the Langmuir oscillation generated by the signal wave in the long (compared to all characteristic times) pulse limit

$$E_l \sim E_0\omega_0/\nu \sim 2.7 \times 10^5\mathrm{V/m}. \tag{71}$$

As we see, the field of the Langmuir oscillation generated by the signal wave is nearly 500 times greater than the level of noise. Therefore the Raman signal can be efficiently measured despite its relatively low absolute amplitude.

Let us estimate the electromagnetic wave intensity needed to produce wave breaking. For the wavelength $\lambda \sim 10$ cm and $k_0L \sim 1$, the threshold power of the electromagnetic wave is

$$P \sim 1.6 \times 10^9(\nu/\omega_0)^4\mathrm{W/cm^2}. \tag{72}$$

At the same time, the quiver velocity must exceed the thermal one. This will be so when the power $P$ exceeds

$$P \sim 3 \times 10^3 (\nu/\omega_0)^2 T[\mathrm{eV}]\mathrm{W/cm}^2. \quad (73)$$

Taking into account that $T \sim 1$ eV and assuming $P = 0.3$ W/cm$^2$, we see that the wave breaking may occur at very moderate ratios, $\nu/\omega_0 \sim 10^{-2}$.

In our calculations we neglected the effect of the geomagnetic field on the resonant absorption and the processes of Brillouin and Raman scattering. This was possible because all frequencies involved in the problem are much higher than the electron gyrofrequency ($\sim 1$ MHz), and the electron Larmor radius ($\sim 7$ cm) is greater than the width of the resonant layer ($\sim \nu L/\omega <$ 1 cm).

In conclusion, let us mention another possibility of communicating through the plasma sheath. As discussed, the linear transformation of the electromagnetic wave generates a potential field amplitude well above the electromagnetic field magnitude. A strong potential field drives a variety of plasma effects which can be observed from the vehicle. On other hand, it is easier to detect the potential field inside the plasma by a Langmuir probe. However, a Langmuir probe can survive in the plasma surrounding the vehicle for several minutes. The size of the probe must be of the order of the plasma thickness, that is $\sim 10$ cm. We would like to emphasize the difference between the detection of the electromagnetic wave, when a big antenna must be located outside the plasma sheath (which is not possible), and the detection of the potential electric field of the plasma wave, for which only a tiny probe inside plasma is needed. Further, if one can place a probe in the resonance zone, then it is possible to measure not only the electric field, but also the magnetic field and the fast electrons, etc. The sensitivity of such measurements is fairly good (see, *e.g. S. C. Cowley et al.*, (preprint, 1993). Bitlike coding of the information makes the problem of the measurement accuracy nonessential (one need only detect the presence/absence of the signal)Availability of several parallel channels and even probes allows a great reduction in the noise level.

We have shown in this paper that even for fixed plasma parameters one has the freedom to choose the wavelength, pulse chirp, the pulse duration, and the interval between the pulses. Changing these parameters changes the relative role of the various channels. Optimization of the communication scheme must be done taking into account both the physical phenomena and the engineering requirements and restrictions.

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## References

Albriton, J., and P. Koch, Cold plasma wavebreaking: Production of energetic electrons, *Phys. Fluids, 18*, 1138, 1975.

Fried, B., and S. Conte, *The Plasma Dispersion Function*, Academic, San Diego, California, 1961.

Gavrilov, E. M., D. F. DuBois, H. A. Rose, and A. M.Rubenchik, Evolution to turbulence in a capacitor model of resonance absorbtion, *Phys. Plasmas, 2*, 1907, 1995.

Ginzburg, V. L., *The Propagation of Electromagnetic Waves in Plasmas*, Pergamon, Tarrytown, N. Y., 1964.

Hinkel-Lipsker, D. E., B. D. Fried, and G. J. Morales, Analytic expressions for mode conversion in a plasma with a linear density profile, *Phys. Fluids B, 4*, 559, 1992.

Kruer, W. L., *The Physics of Laser Plasma Interaction*, Addison-Wesley, Reading, Mass., 1988.

Landau, L. D. and E. M. Lifshitz, *Fluid Mechanics*, Pergamon, Tarrytown, N. Y., 1987.

Lee, A., Y. Nishida, N. C. Luhmann, C. Randall, M. Rhodes and S. P. Obenchain, Resonance absorption in a microwave plasma interaction, *Phys. Fluids B, 29*, 3785, 1986.

Nazarenko, S. V., A. C. Newell, and V. E. Zakharov, Communication through plasma sheaths via Raman (three-wave) scattering process, *Phys. Plasmas, 1*, 2827, 1994.

Nazarenko, S. V., A. C. Newell, and A. M. Rubenchik,On the resonant absorption of a short pulses of electromagnetic radiation, *Phys. Lett. A, 197*, 159, 1995.

Papa, R. J. and R. L. Taylor, High-power electromagnetic transmission characteristics of a diffusing reentry plasma, *J. Appl. Phys., 45*, 684, 1974.

Sagdeev, R. Z., V. D. Shapiro, and V. I. Shevchenko, Anomalous dissipation of strong electromagnetic waves in a laser plasma in *Handbook on Plasma Physics Vol.3*, edited by A. Rubenchik and S. Witkovski, pp. 271-335, Elsevier Science, New York, 1991.

Zakharov, V.E., Collapse and self-focusing of Langmuir waves in *Handbook on Plasma Physics V.2,* edited by A.Galeev and R.Sudun, pp.81-122, Elsevier Science, New York, 1984.

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