Nonlocal turbulence of drift waves

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On the basis of an analysis of weak-turbulence Kolmogorov spectra, it is suggested that drift-wave turbulence is nonlocal: The dominant interaction is an interaction with a zonal flow, rather than a neighboring-scale interaction. The equation for nonlocal drift turbulence is derived. Its physical consequences are analyzed. In particular, two new effects are found. First, as the medium evolves, the k-space spectrum of the drift-wave turbulence separates into two unconnected components: an intense zonal flow and a jet of small-scale turbulence concentrated on an unclosed curve. Second, the very existence of a small-scale weak turbulence fixes the level of the zonal-flow turbulence.

1. INTRODUCTION

1.1. In this paper we analyze the turbulence of drift waves or Rossby waves which have the dispersion relation

$$\omega=\omega_k=\beta\frac{k_x}{1+\rho^2k^2} \qquad (1.1)$$

[$k=(k_x,k_y)$ is the wave vector and β and ρ are constants] and which interact with each other nonlinearly. Since the dispersion relation (1.1) is of a decay nature, the nonlinear interaction of the waves is incorporated in lowest order in the quadratic term on the right side of the dynamic equation for the wave amplitudes $a_k(t)$:

$$i\dot{a}_k=\omega_k a_k+\operatorname{sign}\omega\int V_{-k,k_1,k_2}\delta(k-k_1-k_2)a_{k_1}a_{k_2}dk_1\,dk_2, \qquad (1.2)$$

$$a_k=\overset{*}{a}_{-k}. \qquad (1.3)$$

The matrix element V_{k,k_1,k_2} characterizes the nonlinear wave interaction, which may take different forms. This matrix element has the symmetries

$$\overset{*}{V}_{-k,-k_1,-k_2}=V_{k,k_1,k_2}=V_{kk_2k_1}=V_{k_1,k,k_2}=V_{k_2,k_1,k}. \qquad (1.4)$$

The relations (1.4) mean that the variables $a_k(t)$ are canonical Hamiltonian variables (see Appendix A).

1.2. The most important physical example which leads to Eq. (1.2) is the system of waves described by the Charney-Hasegawa-Mima equation[1,2]

$$\frac{\partial}{\partial t}(\rho^2\Delta\Psi-\Psi)-\beta\frac{\partial\Psi}{\partial x}+A\left(\frac{\partial\Psi}{\partial x}\frac{\partial\Delta\Psi}{\partial y}-\frac{\partial\Delta\Psi}{\partial x}\frac{\partial\Psi}{\partial y}\right)=0 \qquad (1.5)$$

(A = const). If we take Fourier transforms in this equation, switch to the canonical variables $a_k(t)$ (see Ref. 3 and also Appendix A of the present paper), and discard terms of higher than second order in the nonlinearity, we find Eq. (1.2) with the matrix element[3]

$$V_{k,k_1,k_2}=-\frac{iA}{4\pi}|\beta k_x k_{1x}k_{2x}|^{1/2}\left(\frac{k_{1y}}{1+\rho^2k_1^2}+\frac{k_{2y}}{1+\rho^2k_2^2}+\frac{k_y}{1+\rho^2k^2}\right) \qquad (1.6)$$

[see Appendix A regarding the normalization of the Fouri transformation and the choice of variables $a_k(t)$]. Equatic (1.5) arises in studies of phenomena differing in physic nature:

1. Rossby waves in the atmospheres and oceans of rota ing planets.[1,4] In this case, $\rho=(gh_0)^{1/2}/f$ is the Rossb radius,

$$\beta=f\rho^2\left(\frac{\partial}{\partial y}\ln\frac{h_0}{f}\right)_{y=0}$$

is the Rossby velocity, $\Psi=(h_0-h)/h_0$ is the relative pe turbation of the height of the atmosphere or the ocean, an $A=f\rho^4$ (f is the Coriolis parameter, h_0 and h are respectiv ly the equilibrium height and perturbed height, and g is tl acceleration due to gravity).

2. Drift waves in an inhomogeneous magnetized pla ma.[2,5,6] In this case $\rho=(T_e/m_i)^{1/2}/\omega_{Bi}$ is the ion gyrorad us calculated from the electron temperature T_e,

$$\beta=\omega_{Bi}\rho^2\left(\frac{\partial}{\partial y}\ln\frac{n_0}{\omega_{Bi}}\right)_{y=0}$$

is the drift velocity, $\Psi=e\Phi/T_e$, and $A=\omega_{Bi}\rho^4$ (m_i and ω are the ion mass and gyrofrequency, respectively, n_0 is tl equilibrium plasma density, e is the charge of an electroı and Φ is the electric potential).

3. (Lower hybrid)-drift waves in the plasmas of con pact tori, reversed-field pinches, and the ionospheric F la er,[7] for which $\rho=(T_i/m_e)^{1/2}/\omega_{Be}$ is the electron gyrorad us calculated from the ion temperature T_i,

$$\beta=\omega_{Be}\rho^2\left(\frac{\partial}{\partial y}\ln\frac{n_0}{\omega_{Be}}\right)_{y=0}, \qquad \Psi=e\Phi/T_i, \qquad A=\omega_{Be}\rho^4,$$

and m_e and ω_{Be} are respectively the mass and gyrofrequenc of the electrons.

4. Electromagnetic electron oscillations of an inhomc geneous magnetized plasma. These oscillations are seen in pinches and other high-current pulsed discharges.[8] In thi case, $\rho=c/\omega_p$ is the skin depth,

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$$\beta=\omega_{Be}\rho^2\left(\frac{\partial}{\partial y}\ln\frac{n_0}{\omega_{Be}}\right)_{y=0},\quad \Psi=\frac{4\pi n_0 e\Phi}{B_0^2},\quad A=\omega_{Be}\rho^4,$$

c is the velocity of light, ω_p is the plasma frequency, and B_0 is the equilibrium magnetic field.

5. Trapped-ion modes in a tokamak.[9,10] In this case we have $\rho = (T_e/m_i)^{1/2}/\omega_{Bi}\varepsilon$ (ε is the ratio of the minor radius of the tokamak to its major radius),

$$\beta=\omega_{Bi}\rho^2\left(\frac{\partial}{\partial y}\ln\frac{n_0}{\omega_{Bi}}\right)_{y=0},\quad \Psi=e\Phi/T_e,\quad A=\omega_{Bi}\rho^4.$$

6. Density waves in the gas disks of galaxies.[11] In this case Ψ is the gravitational potential, and

$$\beta/\rho^2=2\Omega\frac{\partial}{\partial y}\left(\ln\frac{u}{\Omega}\right)_{y=0},\quad A/\rho^2=1/2\Omega,$$

where Ω is the rotation frequency of the gas disk, and u is the unperturbed density of the galactic gas.

The nonlinearity in Eq. (1.5) is called a "vector nonlinearity." Under the condition $k^2\rho^2 \ll 1$, a "scalar nonlinearity" may also be important in the situations listed above. In such a case, Eq. (1.5) acquires a term $-B\Psi(\partial\Psi/\partial x)$, where $B = \text{const}$. For electron drift waves, for example, we would have[12]

$$B=\omega_{Bi}\rho^2\left(\frac{\partial \ln T_e}{\partial y}\right)_{y=0}.$$

The matrix element of the scalar nonlinearity is[33]

$$V_{k,k_1,k_2}=\frac{B}{4\pi}\,|\beta k_x k_{1x} k_{2x}|^{1/2}. \tag{1.7}$$

The ion drift waves in a magnetized plasma with an inhomogeneous pressure can also be classified as drift waves.[13] Their dispersion relation is

$$\omega_k=\beta k_x\left(1-\rho_i^2k^2+\frac{1}{\rho_g^2k^2}\right), \tag{1.8}$$

where β is the drift velocity along the pressure gradient, and $\rho_i^2k^2$ and $1/(\rho_g^2k^2)$ are small dispersive increments associated with respectively the finite ion Larmor radius and the finite gravitational force. In the case $1\gg(\rho_i k)^2\gg(\rho_g k)^{-2}$ the dispersion relation (1.8) is found from (1.1) through a Taylor-series expansion. In this case we have[13]

$$V_{k,k_1,k_2}=\text{const}\,|k_xk_{1x}k_{2x}|^{1/2}\frac{k^2+k_1^2+k_2^2}{|kk_1k_2|}(k_{1y}k_1^2+k_{2y}k_2^2+k_yk^2). \tag{1.9}$$

In the case $1\gg(\rho_g k)^{-2}\gg(\rho_i k)^2$, we can discard the nondispersive term βk_x from the kinetic equation for the waves from which we start (Sec. 2), although the dispersion relation is very different from (1.1). We can assume that the dispersion is of the form $\beta k_x/(\rho_g k)^2$, which becomes the same as (1.1) under the condition $\rho^2k^2\gg 1$. In this region of parameter values we have[13]

$$V_{k,k_1,k_2}=\text{const}\,|k_xk_{1x}k_{2x}|^{1/2}\frac{k^2+k_1^2+k_2^2}{|kk_1k_2|}\left(\frac{k_y}{k^2}+\frac{k_{1y}}{k_1^2}+\frac{k_{2y}}{k_2^2}\right). \tag{1.10}$$

1.3. It is frequently assumed that only the interaction between waves (or vortices) of approximately the same scale is important in turbulence; i.e., it is assumed that the turbulence is "local." In the present paper, on the basis of a preliminary analysis (Sec. 2), we suggest that drift-wave turbulence is nonlocal. Specifically, we suggest that the evolution of the turbulence is determined primarily by the interaction with turbulence of much lower frequency, rather than with turbulence characterized by frequencies ω_k and wave vectors k which are of approximately the same scale. More precisely, the behavior of the turbulence is governed primarily by the interaction with the zonal flow, i.e., by turbulence characterized by very small values of k_x and ω_k.

The primary result of this study is the discovery that the following two effects (Sec. 3) occur if this hypothesis of nonlocal nature is correct. (1) "Intermediate scales die out": As time elapses, the turbulence spectrum in k space splits into two unconnected components. One is a low-frequency turbulence of the zonal flow, and the other is a high-frequency, short-wavelength jet. This jet is concentrated along a curve $\omega_k - \beta k_x = \text{const}$. Throughout the remainder of k space, the turbulence spectrum vanishes exponentially. (2) The existence of weak high-frequency turbulence leads to a very restrictive condition on the turbulence of the zonal flow, which may in general be strong. This condition is that a certain global spectral characteristic of the zonal flow must take on a certain fixed value. The rate of energy dissipation in the zonal flow determines the intensity of the jet of small-scale turbulence.

We also show here that the nonlocal interaction with the zonal flow implies that the small-scale turbulence spectrum becomes more symmetric under the reflection $k_y \to -k_y$. A Kolmogorov jet spectrum of high-frequency turbulence corresponding to pumping of quasiparticles toward large values of $|k|$ is obtained.

This paper effectively has two parts, which can largely be read independently. In the first part (Sec. 2), we formulate and provide a foundation for the suggestion that drift-wave turbulence is nonlocal. In the second part (Sec. 3), we work from this suggestion to derive an equation for nonlocal drift-wave turbulence, and we discuss the physical consequences of this equation.

2. HYPOTHESIS OF A NONLOCAL NATURE

2.1. In 1922, Richardson[14] suggested that only the interactions between vortices of approximately the same scale were important in a hydrodynamic turbulence. As a result of such interactions, energy would be transferred step by step from relatively large vortices to relatively small ones. Working from this "hypothesis of the local nature of turbulence" and dimensionality considerations, Kolmogorov and Obukhov[15,16] derived the energy spectrum of the turbulence of an incompressible fluid:

$$\varepsilon_k=CP^{2/3}|k|^{-11/3}, \tag{2.1}$$

(C is a universal dimensionless constant). This spectrum is determined by the energy flux P from scale to scale toward large $|k|$. In 2D hydrodynamics, there is an integral of motion not present in 3D hydrodynamics, the enstrophy. A similar approach leads to two turbulence spectra of an incompressible fluid, as was shown by Kraichnan.[17,18] One of these spectra,

$$\varepsilon_k=C_1P^{2/3}|k|^{-8/3}, \tag{2.2}$$

describes turbulence with an energy cascade, while the second,

$$\varepsilon_k = C_2 Q^{2/3} |k|^{-4}, \tag{2.3}$$

describes turbulence with an enstrophy cascade. The energy flux P is directed toward small values of $|k|$, while the enstrophy flux Q is directed toward large values of $|k|$. The spectrum (2.1) is supported by experimental data, while spectra (2.2) and (2.3) agree with the results of numerical simulations. However, we do not yet have a rigorous derivation of Kolmogorov spectra (2.1)–(2.3). In fact, we do not even have a proof of the hypothesis that hydrodynamic turbulence is local.

Adopting the hypothesis of local nature, and making use of dimensionality considerations, one can construct spectra analogous to spectra (2.1)–(2.3) for a wide range of weak-turbulence media (see Refs. 19 and 20 and the review in Ref. 21). These "Kolmogorov weak-trubulence spectra" describe turbulence in which there is a cascade from scale to scale of some integral of motion or other of the dynamic equations. An important circumstance, which was established by one of the present authors,[19,20] is that in the case of weak turbulence the Kolmogorov spectra can be derived systematically from the equations of the medium, and the hypothesis of local nature can be tested explicitly. Weak-turbulence Kolmogorov spectra are exact steady-state solutions of the wave kinetic equations[22–24]

$$\frac{\partial n}{\partial t} = St[n], \tag{2.4}$$

which are derived by taking an average of the dynamic equations. Here $n = n_k = \varepsilon_k / \omega_k$ is the spectrum of the wave action, ε_k is the energy spectrum, and $St[n]$ is the collision integral. Kuznetsov[25] has derived weak-turbulence Kolmogorov spectra as exact solutions of the kinetic equations for very anisotropic media also. These spectra are power-law functions not of the absolute value of the wave vector (as in the case of isotropic media) but of its components. In the 2D case (to which drift waves belong[26,27]), for example, we have

$$n_k = \mathscr{R} k^{-\nu} \equiv \mathscr{R} |k_x|^{-\nu_x} |k_y|^{-\nu_y}, \quad \nu = (\nu_x, \nu_y), \tag{2.5}$$

where the constant $\mathscr{R}$ is determined by the flux (rate of dissipation) of some integral of motion of the dynamic equations, and the exponent $\nu = (\nu_x, \nu_y)$ is expressed in terms of the properties of the medium.

2.2. A necessary condition for Kolmogorov turbulence to be local is that the collision integral converge in the case of the Kolmogorov spectrum: Only under this condition is the Kolmogorov spectrum a solution of the kinetic equation.[19–21] On the other hand, this condition is not sufficient for localness. It turns out that while converging for the Kolmogorov spectrum itself the collision integral $St[n]$ may diverge for spectra n_k differing from Kolmogorov spectrum n_k^0 by arbitrarily small "finite" perturbations $\delta n_k = n_k - n_k^0$. We say that a perturbation δn_k is "finite" if it is identically zero outside a certain range of scales[1)] [e.g., in the isotropic case, $\delta n_k \equiv 0$ as $|k| \to 0$ and $|k| \to \infty$; alternatively, in an anisotropic 2D medium, $\delta n_k \equiv 0$ if at least one of the conditions $|k_x| \to 0$, $|k_x| \to \infty$, $|k_y| \to 0$, $|k_y| \to \infty$ holds, where $k = (k_x, k_y)$].

Furthermore, the convergence of the collision integral for the Kolmogorov spectrum itself and for spectra which differ from it by small finite perturbations is still not sufficient to guarantee that the turbulence will be local.[28–30] It turns out that if we specify a small finite initial perturbation of a Kolmogorov spectrum[2)] then the evolution of the medium may result in the appearance of a spectrum for which the collision integral diverges.

It is worthwhile to examine the question of whether a Kolmogorov spectrum is local with respect to some class of perturbations or other. A Kolmogorov spectrum which is nonlocal with respect to the class of all perturbations may prove to be local with respect to a narrower class of perturbations. For isotropic media, for example, we should examine the question of whether the Kolmogorov spectrum is local with respect to isotropic perturbations, which obviously remain isotropic as time elapses. For drift waves under the condition

$$|V(k_x, k_y; k_{1x}, k_{1y}; k_{2x}, k_{2y})| = |V(k_x, -k_y; k_{1x}, -k_{1y}; k_{2x}, -k_{2y})|$$

perturbations which are even (i.e., which are symmetric un der the substitution $k_y \to -k_y$) remain so in the course o the evolution, and it is useful to determine whether Kolmo gorov spectra are local with respect to this class of perturba tions. Numerical simulations of turbulence are frequentl restricted to even spectra (to reduce the amount of memor and CPU time required), so one must determine whethe turbulence is local with respect to the class of even perturba tions in order to correctly interpret the results of th calculations.

If the turbulence is nonlocal, a study of its behavic requires consideration of the interaction with distant scale at the edges of the inertial range. In reality, the neighbo hoods of singular points of the collision integrals, at whic divergence occurs in the case of an infinite inertial interv (i.e., in the case of a finite inertial interval), contribute sul stantially to the collision integral. One might say that there a nonlocal interaction with these neighborhoods.

The nonlocal nature of a Kolmogorov spectrum m mean that this spectrum cannot be realized physically. A example is a Kolmogorov spectrum for which the collisi integral diverges. On the other hand, there may also be situ tions in which a Kolmogorov spectrum is nonlocal with r spect to a certain class of perturbations, but the nonloc interaction which arises suppresses these perturbations, sc Kolmogorov spectrum can be realized. For example, loc ness with respect to isotropic perturbations in the isotro case and localness with respect to perturbations which a even in the variable k_y in the case of drift waves are necessa for the occurrence of Kolmogorov spectra in these sit tions. The nonlocal interaction which arises from an ani tropic perturbation component in the isotropic situation a from a perturbation component which is odd in k_y in t case of drift waves results in suppression of these com nents of the perturbations. It does not prevent Kolmogo spectra from occurring (Appendix B and Sec. 3).

It can be shown that the conditions for the converge of a collision integral at small scales for the Kolmogo spectrum itself and for a spectrum differing from it by arbitrary small finite perturbation are identical (this is tr at any rate, for isotropic media and of drift waves). If a K mogorov spectrum is the solution of a kinetic equation, then nonlocalness with respect to finite perturbations cannot be manifested in the interaction with small scales of the turbulence in this case. Conditions under which Kolmogorov spectra are local with respect to various classes of perturbations have been derived in several places[3)] (Refs. 28–32).

2.3. For media which can be described by the dynamic equation (1.2), the kinetic equation (2.4) can be written as follows (see Ref. 23 and Subsection A.4 of Appendix A):

$$\frac{\partial n}{\partial t}=\int 4\pi |V_{k,k_1,k_2}|^2\delta(k+k_1+k_2)\delta(\omega_k+\omega_{k_1}+\omega_{k_2}) \times [n_{k_1}n_{k_2}+n_k n_{k_1}\,\mathrm{sign}(\omega_k\omega_{k_2}) +n_k n_{k_2}\,\mathrm{sign}(\omega_k\omega_{k_1})]dk_1\,dk_2, \tag{2.6}$$

where $n_k=|a_k|^2=n_{-k}$. This equation conserves three integrals: the energy $\mathscr{E}$, the enstrophy (the x momentum) $\mathscr{P}_x$, and the y momentum $\mathscr{P}_y$ [which correspond to the integrals of motion of dynamic equation (1.2); see Subsection A.2 of Appendix A]:

$$\mathscr{E}=\frac{1}{2}\int|\omega_k|n_k\,dk, \tag{2.7}$$

$$\mathscr{P}_x=\frac{1}{2}\int|k_x|n_k\,dk, \tag{2.8}$$

$$\mathscr{P}_y=\frac{1}{2}\int k_y\,\mathrm{sign}\,k_x n_k\,dk \tag{2.9}$$

[the integral (2.9) is zero for all spectra of the type in (2.5)].

To derive the Kolmogorov spectra (2.5), one assumes that the medium is scale-invariant in terms of the components of the vector k:

$$\omega(k)=\mathrm{const}\,|k|^\alpha=\mathrm{const}\,|k_x|^{\alpha_x}|k_y|^{\alpha_y},\quad \alpha=(\alpha_x,\alpha_y);$$
$$V(qk,qk_1,qk_2)=q^\beta V(k,k_1,k_2),\quad \beta=(\beta_x,\beta_y), \tag{2.10}$$

where $q=(q_x,q_y)$ is an arbitrary vector with positive components,

$$(qk)=(q_xk_x,q_yk_y),\quad q^\beta=|q_x|^{\beta_x}|q_y|^{\beta_y}.$$

For a system of drift waves there are then two Kolmogorov spectra:[26,27,32]

$$n=C_1P^{1/2}k^{-\nu},\quad \nu=d+\beta,\quad d=(1,1), \tag{2.11}$$

$$n=C_2Q^{1/2}k^{-\nu},\quad \nu=d+\beta+[(1,0)-\alpha]/2, \tag{2.12}$$

where P and Q are the fluxes (rates of dissipation in the system) of energy and enstrophy, respectively, and C_1 and C_2 are dimensionless constants.

For waves with the dispersion relation (1.1), the scale invariance in (2.10) prevails only under the condition $|k_y|\gg|k_x|$, and then only if either $\rho|k|\gg1$ holds (a short-wavelength turbulence), in which case we have

$$\omega=\frac{\beta}{\rho^2}\frac{k_x}{k_y^2},\quad \alpha=(1,-2), \tag{2.13}$$

or if $\rho|k|\ll1$ holds (long-wavelength turbulence), in which case we have[4)]

$$\omega=\beta k_x(1-\rho^2k_y^2),\quad \alpha=(1,2). \tag{2.14}$$

2.4. Let us examine Kolmogorov spectra and the question of whether they are local for a drift-wave turbulence.[32] For the Charney-Hasegawa-Mima equation [see (1.5) and (1.6)], we have the following result[26,27] in the short-wavelength region:

$$V_{kk_1k_2}=\mathrm{const}\,|k_xk_{1x}k_{2x}|^{1/2}(k_y^{-1}+k_{1y}^{-1}+k_{2y}^{-1}),\quad \beta=(3/2,-1).$$

In this case, a spectrum with an exponent $\nu=(5/2,1)$, corresponding to a pumping of enstrophy [see (2.12)], is not a solution of the kinetic equation (2.6), and the collision integral diverges for this spectrum at both large and small wave numbers. A Kolmogorov spectrum with an exponent $\nu=(5/2,0)$, corresponding to pumping of energy [see (2.11)], is a solution of the kinetic equation (2.6), since the collision integral converges for this spectrum. For any finite perturbations of the spectrum, however, the collision integral diverges at large scales ($|k_1|\to0$), while for finite perturbations which have an odd part the collision integral also diverges as $k_1\to(0,-2k_y)$. For a long-wavelength turbulence we have

$$V_{kk_1k_2}=\mathrm{const}\,|k_xk_{1x}k_{2x}|^{1/2}(k_y^3+k_{1y}^3+k_{2y}^3),\quad \beta=(3/2,3).$$

A Kolmogorov spectrum with an energy flux [$\nu=(5/2,4)$] is a solution of kinetic equation (2.6), but for any finite perturbations of this spectrum the collision integral diverges at the singular point $k_1=(0,0)$. If there is a component of the perturbation which is odd with respect to k_y, the collision integral also diverges at the singular point $k_1=(0,-2k_y)$. A Kolmogorov spectrum with an enstrophy flux [$\nu=(5/2,3)$] is a solution of the kinetic equation (2.6), but for perturbations with a nonzero odd component the collision integral diverges at the point $k_1=(0,-2k_y)$.

For a scalar nonlinearity [$\alpha=(1,2),\beta=(3/2,0)$; see (2.14) and (1.7)], both of these Kolmogorov spectra [$\nu=5/2,1$), with a flux P, and $\nu=(5/2,0)$, with a flux Q; Ref. 33] are solutions of the kinetic equation (2.6), but for perturbations which have a component which is odd in k_y the collision integral diverges as $k_1\to(0,2k_y)$. Furthermore, a Kolmogorov spectrum with an enstrophy flux is nonlocal (or highly unstable[30,32]) with respect to the evolution of even perturbations.

For ion drift waves,[13] in the case in which the dispersion stems from a finite Larmor radius $1\gg(\rho_ik)^2\gg(\rho_gk)^{-2}$; $|k_y|\gg|k_x|$, we have $\alpha=(1,2),\beta=(3/2,2)$ [see (1.8) and (1.9)]. A Kolmogorov spectrum with an enstrophy flux [$\nu=(5/2,2)$] is not a solution of the kinetic equation (2.6) (for such a spectrum, the collision integral diverges as $|k_1|\to0$ and also as $|k_1|\to\infty$). Although a spectrum with an energy flux [$\nu=(5/2,3)$] is a solution of the kinetic equation, the collision integral diverges at small wave numbers ($|k_1|\to0$) or for any finite perturbations of this spectrum. If there are perturbations which are odd with respect to k_y, there is also a divergence as $k_1\to(0,-2k_y)$.

If the dispersion of ion drift waves is a consequence of a gravitational force $1\gg(\rho_gk)^{-2}\gg(\rho_ik)^2$; $|k_y|\gg|k_x|$, we have $\alpha=(1,-2),\beta=(3/2,-2)$. In this case a spectrum with an energy flux [$\nu=(5/2,-1)$] is not a solution of the kinetic equation, and the collision integral diverges as $|k_1|\to0$ and also as $|k_1|\to\infty$. A spectrum with an enstrophy flux [$\nu=(5/2,0)$] is a solution of the kinetic equation (2.6), but if there are finite perturbations of any shape the

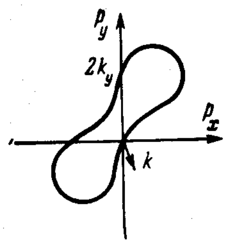

FIG. 1. Characteristic shape of the resonance curve, which determines pairs of vectors k_1, k_2 which, along with the given vector, satisfy the equations $k + k_1 + k_2, \omega_k + \omega_{k_1} + \omega_{k_2} = 0$. Each point p on this curve specifies two solutions of these equations: $k_1 = p$, $k_2 = -k - p$ and $k_2 = p$, $k_1 = -k - p$ (Ref. 1).

collision integral diverges as $|k_1| \to 0$. In addition, this spectrum (like all the spectra listed above) is nonlocal with respect to odd perturbations.

We see that anisotropic Kolmogorov spectra are usually nonlocal, in contrast with isotropic Kolmogorov spectra.[21,30] These examples indicate that drift-wave turbulence may be nonlocal: The evolution of the turbulence (characterized by scales smaller than a certain value) may be determined primarily by the interaction of the turbulence with turbulence of much larger scale (rather than of approximately the same scale). In these situations, Kolmogorov spectra frequently have the property that the collision integral either diverges as $|k_1| \to 0$ for the Kolmogorov spectrum itself or does so for a spectrum which differs from a Kolmogorov spectrum by an arbitrarily small finite perturbation.

Figure 1 shows the characteristic shape of the resonant manifold [see (A.9)], along which the integration is carried out in the collision integral in Eq. (2.6). The nonlocal nature of the drift-wave turbulence, which is manifested in a strong interaction with large scales, means that the region of the resonance curve near the null vector contributes substantially to the collision integral.

According to the above analysis of Kolmogorov spectra, the region of the resonance curve near the point $(0, -2k_y)$ can also contribute substantially to the collision integral. For all the cases listed above, the collision integral diverges as $k_1 \to (0, -2k_y)$ for spectra which differ from a Kolmogorov spectrum by arbitrarily small finite perturbations which have a component odd in k_y [in the situations which we have been discussing, this nonlocal nature prevails in general for all power-law solutions of the type (2.5) with $\nu_x > 2$; for Kolmogorov spectra (2.11) and (2.12) we have $\nu_x = 2.5$].

Let us assume that the regions of the resonance curve near the points (0,0) and $(0, -2k_y)$ dominate the collision integral, so we can ignore the remainder of the resonance curve. In other words, we assume that the drift-wave turbulence is nonlocal and that its behavior is determined primarily by the interaction with the zonal flow, i.e., with turbulence characterized by wave vectors with small components p_x (and thus low frequencies ω_p).

Some of the Kolmogorov spectra discussed above exhibit a nonlocal nature not only near the zonal flow but also at large scales. In such a situation, one might expect the appearance of a significant nonlocal interaction with substantially smaller scales [but note that for turbulence with $\rho|k| \sim 1$, $\rho|k_x| \sim 1$ a nonlocal interaction with small-scale turbulence would not be possible, since the resonant manifold (Fig. 1) does not pass through wave vectors p with absolute values $|p| \gg |k|$]. The question of nonlocal interaction specifically with the zonal flow, which is indicated indirectly by several experimental facts, seems to us to be the most important question here. Intense zonal flows are not rare in the atmospheres of rotating planets, e.g., that of Jupiter.[34,5] Observations of the spectrum of a plasma turbulence in the F layer of the equatorial ionosphere show that in the range of scales corresponding to drift waves ($5\ \text{m} \lesssim |k|^{-1} \lesssim 100\ \text{m}$; in this case we have $\rho \sim 5$ m) the n_k spectrum has a $|k|^{-6.5}$ behavior.[35] Since the collision integral diverges for power-law spectra of the type in (2.5) as $|k| \to 0$, in which case we have $\nu_x + \nu_y > 6$ (in the case of a vector nonlinearity) or $\nu_x + \nu_y > 4$ (in the case of a scalar nonlinearity), these observations indicate the possibility of a nonlocal interaction with large scales. Numerical simulations[5,36,37] also point to a concentration of the spectrum in the region of zonal flows. Yet another circumstance which is evidence of a nonlocal interaction with the zonal flow is discussed in the following section of this paper.

3. EVOLUTION OF A NONLOCAL TURBULENCE

3.1. We thus assume that drift-wave turbulence is non local. Specifically, the behavior of this turbulence is deter mined primarily by the interaction with the zonal flow. Th kinetic equation (2.6) then reduces to

$$\frac{\partial n_k}{\partial t} = 2 \int_{\rho|p_x| \ll \rho|k_x|, 1} 4\pi |V_{k,p,-k,-p}|^2 \delta(\Omega_{k+p} - \Omega_k - \Omega_p) \times (n_{k+p} - n_k) n_p \, dp \qquad (3.1$$

(we are assuming that the vector k does not lie in the regio of the zonal flow). We have ignored the term $n_k n_{k+p}$, und the assumption that n_p is quite large. In Eq. (3.1) we use the function

$$\Omega_k = \beta k_x - \omega_k = \frac{\beta k_x \rho^2 k^2}{1 + \rho^2 k^2},$$

which plays an important role in the discussion below. T quantity $-\Omega_k$ represents the frequency of the wave in t coordinate system moving at the drift velocity β.

The resonance curve in Fig. 1 intersects the region the zonal flow in neighborhoods of two points: (0,0) a $(0, -2k_y)$. Let us examine the contribution of each of the neighborhoods to the collision integral.

Denoting the integrand in (3.1) by $F(k,p)$,

$$F(k, p) = 4\pi |V_{k,p,-k-p}|^2 \delta(\Omega_{k+p} - \Omega_k - \Omega_p)(n_{k+p} - n_k) n_p$$

and noting that we have $F(k,p) = -F(k+p, -p)$, we fi the contribution to the collision integral from the neighb hood of the first point, $p = (0,0)$:

$$\int_{|p| \ll |k|, 1/\rho} [F(k,p) - F(k-p,p)] \, dp = \int_{|p| \ll |k|, 1/\rho} \frac{\partial F(k,p)}{\partial k_\alpha} p_\alpha \, dp = \frac{\partial}{\partial k_\alpha} \mathscr{D}_{\alpha\beta} \frac{\partial n}{\partial k_\beta}, \qquad (3$$

$$\mathscr{D}_{\alpha\beta} = \int 4\pi |V_{k,p,-k-p}|^2 \delta(\Omega_{k+p} - \Omega_k - \Omega_p) n_p p_\alpha p_\beta \, dp. \qquad (3$$

Expression (3.2)–(3.3) was derived by Vedenov and Ru kov[38] in a description of the diffusion of plasmons in a random field of ion acoustic waves. It follows from the resonance condition $\Omega_{k+p} - \Omega_k - \Omega_p = 0$ that asymptotically at small values of $|p|$ we have $p_x = \varphi p_y$, where

$$\varphi = \frac{\partial\Omega}{\partial k_y} \Big/ \frac{\partial\Omega}{\partial k_x} = \frac{2k_x k_y}{k^2(1+\rho^2k^2)+2k_x^2}. \tag{3.4}$$

Making use of the δ-functions of the frequencies to integrate over p_x in (3.3), we find

$$\|\mathscr{D}_{\alpha\beta}\| = S\frac{\partial\Omega}{\partial k_x}\left\|\begin{matrix}\varphi^2 & -\varphi\\ -\varphi & 1\end{matrix}\right\|, \tag{3.5}$$

$$S = \left(\frac{\partial\Omega}{\partial k_x}\right)^{-2}\int_{-\infty}^{+\infty} 4\pi[\,|V_{k,p,-k-p}|^2 n_p]_{p_x=-\varphi p_y} p_y^2\, dp_y. \tag{3.6}$$

The tensor in (3.5) is degenerate. Using the change of variables $(k_x, k_y) \to (k_y,\ v = \Omega(k))$ we can reduce (3.2) to an expression which contains only differentiation with respect to k_y:

$$\frac{\partial\Omega}{\partial k_x}\frac{\mathscr{D}}{\mathscr{D}k_y} S \frac{\mathscr{D}}{\mathscr{D}k_y} n.$$

Here and below, $\mathscr{D}/\mathscr{D}k_y$ means differentiation with respect to k_y at constant $v = \Omega(k)$:

$$\frac{\mathscr{D}}{\mathscr{D}k_y} = -\varphi\frac{\partial}{\partial k_x} + \frac{\partial}{\partial k_y}; \qquad n = n(k_y, v).$$

Near the second point, $p = (0, -2k_y)$, integration over p_y in (3.1) with the help of the δ-functions of the frequencies leads to the expression

$$\frac{\partial\Omega}{\partial k_x} Y[n(-k_y, v) - n(k_y, v)],$$

where

$$Y = \left|\frac{\partial\Omega}{\partial k_x}\frac{\partial\Omega}{\partial k_y}\right|^{-1} \times \int_{-\infty}^{+\infty} 8\pi|V(k_x, k_y; p_x, -2k_y; -k_x, k_y)|^2 n(p_x, -2k_y)\, dp_x. \tag{3.7}$$

It is easy to see that $Y(k_y, v)$ is an even function of k_y.

In determining S and Y, we replaced the finite integration limits by infinite limits, since the assumption that the turbulence is nonlocal makes the collision integral negligible outside the neighborhoods of the points (0,0) and $(0, -2k_y)$.

The kinetic equation (3.1) thus becomes

$$\frac{\partial n}{\partial t} = \frac{\partial\Omega}{\partial k_x}\left\{\frac{\mathscr{D}}{\mathscr{D}k_y} S \frac{\mathscr{D}}{\mathscr{D}k_y} n + Y[n(-k_y, v) - n(k_y, v)]\right\} + \gamma n, \tag{3.8}$$

where we have added a term γn, which describes the sources and sinks $[\gamma = \gamma(k_y, v)]$.

3.2. According to Eq. (3.8), the evolution of the spectrum $n(k_y, v)$ in k space occurs independently on each of the curves

$$\Omega(k) = v = \text{const}, \tag{3.9}$$

which are shown in Fig. 2. The redistribution of the turbulence among curves is a slower process, determined by corrections to Eq. (3.8) which may very well be local, described by an integral operator rather than by a differential-difference operator.

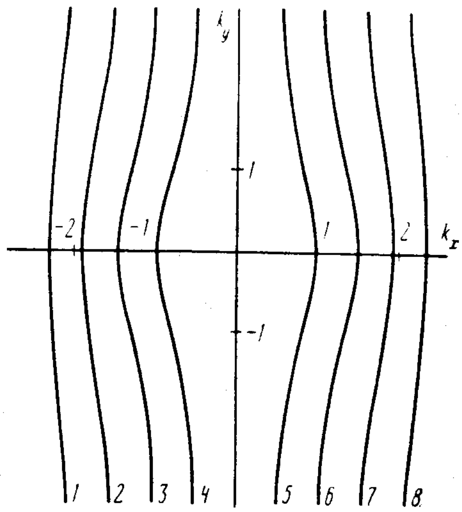

FIG. 2. Curves of $\Omega(k) = v =$ const, along which small-scale waves interact in k space. *1* — $v = -2$; *2* — -1.5; *3* — -1; *4* — -0.5; *5* — 0.5; *6* — 1; *7* — 1.5; *8* — 2.

On each of curves (3.9), Eq. (3.8) conserves the number of particles in the case $\gamma = 0$:

$$N(v) = \int n\delta(\Omega_k - v)\, dk, \qquad \dot{N} = \int \gamma n\delta(\Omega_k - v)\, dk. \tag{3.10}$$

It can be seen from (3.10) that if the quantity γ were positive everywhere on some curve (3.9) then the total number of particles on this curve would increase without bound. Equation (3.8) would then be insufficient (for finding stationary states, for example), since it would not reflect the leakage of wave action from the curve. Such a situation might arise if the curves along which the interaction occurred (in lowest order) were bounded (Appendix B). If we wish to ignore the following order and to limit the discussion to Eq. (3.8), we must require that there be an effective sink on each curve (more on this below). For drift waves, the curves (3.9) are not closed; they run off to infinity, where there is always a strong viscous dissipation.

For spectra which are even in k_y, the second term in braces in (3.8) vanishes. This result agrees with the circumstance that this term arose when we incorporated the evolutionary nonlocal nature of the turbulence with respect to perturbations having an odd component [manifested near the point $(0, -2k_y)$]. The integral in (3.7) diverges at the point $p_x = 0$ for all power-law spectra of the type in (2.5) with $\nu_x > 2$, i.e., whenever this nonlocal nature prevails (Sec. 2). It is also simple to verify that the integral in (3.6) diverges at the point $p_y = 0$ for all Kolmogorov spectra which are nonlocal at large scales (Sec. 2).

The conclusion that drift-wave turbulence is nonlocal—the conclusion which leads to Eq. (3.8)—was reached on the basis of an analysis of whether Kolmogorov spectra known only in the sector $|k_y| \gg |k_x|$ are local. Nevertheless, Eq. (3.8) can also be examined outside this sector. The reason is that the quantity $Y(k)$ is determined by the spectrum n_p, for which we have $|p_y| = 2|k_y| \gg |p_x|$ [see (3.7)]. The quantity $S(k)$ is determined by the spectrum n_p on the straight line $p_x = \varphi p_y$. The quantity $\varphi(k)$ is small not only for $|k_y| \gg |k_x|$ but also in other situations [e.g., $|k_y| \ll |k_x|$ or $\rho^2 k^2 \gg 1$; in any case we have $|\varphi(k)| < 1/\sqrt{3}$].

3.3. Let us examine the time evolution of the energy and the momentum of each of curves (3.9):

$$\mathscr{E}(v) = \frac{1}{2}\int |\omega_k| n_k \delta(\Omega(k) - v)\,dk,$$

$$\mathscr{P}(v) = \frac{1}{2}\int k \operatorname{sign} k_x n_k \delta(\Omega(k) - v)\,dk. \qquad (3.11)$$

According to Eq. (3.8) we have

$$2\dot{\mathscr{E}}(v)\operatorname{sign} v = -\beta\int_{-\infty}^{+\infty} \varphi Q\,dk_y + \int_{-\infty}^{+\infty} \omega_k \gamma_k n_k \left(\frac{\partial\Omega}{\partial k_x}\right)^{-1} dk_y, \qquad (3.12)$$

$$2\dot{\mathscr{P}}_x(v)\operatorname{sign} v = -\int_{-\infty}^{+\infty} \varphi Q\,dk_y + \int_{-\infty}^{+\infty} k_x \gamma_k n_k \left(\frac{\partial\Omega}{\partial k_x}\right)^{-1} dk_y, \qquad (3.13)$$

$$2\dot{\mathscr{P}}_y(v)\operatorname{sign} v = \int_{-\infty}^{+\infty} Q\,dk_y + \int_{-\infty}^{+\infty} k_y Y[n(-k_y, v) - n(k_y, v)]\,dk_y + \int_{-\infty}^{+\infty} k_y \gamma_k n_k \left(\frac{\partial\Omega}{\partial k_x}\right)^{-1} dk_y, \qquad (3.14)$$

where $Q(k_y, v) = -S\mathscr{D}n_k/\mathscr{D}k_y$ is the diffusion flux of particles along the curve. The integration here is along the curve (3.9); i.e., it is assumed that k_x is a function of k_y, specified implicitly by the equation $\Omega(k_x, k_y) = v$. Since the energy and the momentum are integrals of motion of the entire system in the case $\gamma = 0$, the first terms in expressions (3.12)–(3.14), taken with a minus sign, give us the energy and momentum fluxes from the curve to the zonal flow. Equations (3.12)–(3.14) have a clear physical meaning: The number of particles is conserved on each of curves (3.9) [see (3.10)]. As they move (in a diffusive fashion) along the curve (3.8) from the source at small $|k_y|$ to the dissipation region at large $|k_y|$ (Fig. 2), the quanta go through states with smaller values of $|\omega_k|$ and $|k_x|$ but larger values of $|k_y|$. They thus lose energy and enstrophy, but they acquire y momentum.

The energy and enstrophy which are lost are transferred to the large-scale turbulence of the zonal flow, causing an increase in the spectrum at points p such that $p_x = -\varphi(k_y, v)p_y$ (k_y is the instantaneous coordinate of the particle on the curve). At these points, y momentum is acquired which is equal in magnitude and opposite in sign to the momentum acquired by the particle on the curve. If the spectrum on the curve (3.9) is not even in k_y, the y momentum is transferred to the zonal flow in a different way [see the second integral in (3.14)], without an accompanying transfer of energy and enstrophy.

3.4. For a given spectrum n_p of the turbulence in the zonal flow, the quantities S and Y are functions of k_y and v. The nature of the evolution of the spectrum along the curve (3.9) is then determined by the solution of the eigenvalue $[\lambda(v)]$ problem

$$\lambda n = \frac{\partial\Omega}{\partial k_x}\left\{\frac{\mathscr{D}}{\mathscr{D}k_y} S \frac{\mathscr{D}}{\mathscr{D}k_y} n + Y[n(-k_y, v) - n(k_y, v)]\right\} + \gamma n \qquad (3.15)$$

with the boundary conditions

$$n(+0, v) = n(-0, v),$$
$$Q(+0, v) = Q(-0, v), \quad Q = o(1/k_y), \quad k_y \to \pm\infty. \qquad (3.16)$$

Equation (3.15) obviously reduces to a system of two second-order differential equations. For spectra which are even in k_y, Eq. (3.15) is itself a second-order differential equation,

$$\lambda n = \frac{\partial\Omega}{\partial k_x}\frac{\mathscr{D}}{\mathscr{D}k_y} S \frac{\mathscr{D}}{\mathscr{D}k_y} n + \gamma n, \qquad (3.17)$$

which is to be solved under the boundary conditions

$$Q(0, v) = 0, \quad Q = o(1/k_y), \quad k_y \to +\infty. \qquad (3.18)$$

The eigenvalues depend on the function $\gamma(k_y, v)$ (on the shape of the "potential well"). For drift waves in various physical situations, the functions $\gamma(k_y, v)$ turn out to be approximately the same (see Refs. 7 and 35–37). A typical shape of this function is shown by the contour plot in Fig. 3. As a rule, the function reaches a maximum on the k_x axis, and the $\gamma = 0$ contour line frequently passes through the point $k = 0$ (Refs. 7 and 36).

As time elapses, the spectrum on the curve (3.9) tends toward an eigenfunction corresponding to the maximum eigenvalue $\bar{\lambda}(v)$. This eigenfunction is positive everywhere. If, at some v, we have $\bar{\lambda}(v) > 0$, the spectrum on curve (3.9) increases exponentially with the time: The source creates more particles than are carried off diffusively to the dissipation region. If $\bar{\lambda}(v) < 0$, the spectrum $n(k_y, v)$ tends toward zero: The diffusion coefficient is so large on the curve (3.9) that the particles move off into the dissipation region faster than they are created by the source.

Accordingly, if there exist values of v for which the condition $\bar{\lambda}(v) > 0$ holds [such values of v correspond to curves (3.9) with large growth rates $\gamma(k_y, v) > 0$], then Eqs (2.12)–(2.14) tell us that the energy and momentum fluxes into the large-scale turbulence of the zonal flow will increase with time, causing an increase in the spectrum of the zonal flow turbulence. We would naturally expect that this process would lead to an increase in the diffusion coefficient S and thus the rate of the diffusive escape of particles to the dissipation region, i.e., to a decrease in the eigenvalues $\bar{\lambda}(v)$. If the curve passes through a region of strong dissipation, the eigenvalue $\bar{\lambda}(v)$ always goes negative for a sufficiently large increase in S. The interval of values of v for which the cond-

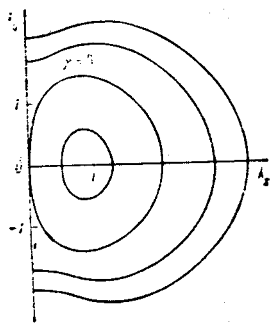

FIG. 3. Typical appearance of contour lines of the function $\gamma(k)$. T growth rate ($\gamma > 0$) is in a region adjacent to the k_x axis, reaching a ma mum on this axis; we have $\gamma \to -\infty$ as $|k| \to \infty$.

tion $\bar{\lambda}(v) > 0$ holds thus shrinks as time elapses, until it degenerates into a point v_0, with $\bar{\lambda}(v) = 0$ [the source is in a sense at a maximum on the $\Omega(k) = v_0$ curve]. The k-space turbulence spectrum thus splits up into two unconnected components: a zonal flow and a high-frequency, short-wavelength jet concentrated on a curve $\Omega(k) = v_0$ ("intermediate scales die out").

3.5. It can be concluded from this discussion that a small-scale weak turbulence imposes a severe limitation on the turbulence of the zonal flow. To illustrate this point, we consider an example. We assume that the turbulence spectrum is even in k_y. At small $|p|$, the asymptotic formula

$$[|V_{k,p,-k-p}|^2]_{p_x=-\varphi p_y} = p_y^u U(k),$$

with some number u and some function $U(k)$, usually holds. The integral in (3.6) can then be written in the form

$$S = \mu f(k_y, v),$$

where

$$f(k_k, v) = 4\pi \left(\frac{\partial \Omega}{\partial k_x}\right)^{-2} U(k), \quad \mu = \int_{-\infty}^{+\infty} n(-\varphi p_y, p_y) p_y^{u+2} dp_y. \tag{3.19}$$

The value of $\mu(k)$ at points on the curve (3.9) depends on the shape of the spectrum in the sector

$$|p_x/p_y| < \varphi_{max}(v) = \max_{-\infty < k_y < +\infty} |\varphi(k_y, v)| \tag{3.20}$$

alone. This sector is comparatively narrow: $\varphi_{\max}(v) \to 0$ as $v \to \infty$ (Fig. 4). We assume that at $v = v_0$ the sector (3.20) is so narrow that in it we can ignore the angular dependence of the large-scale turbulence spectrum n_p (we are assuming that the spectrum n_p has no singularities on the k_y axis). The quantity (3.19) is then independent of k. The point v_0 is the value of v for which, at the maximum value of the parameter μ, there exists a positive solution of the equation

$$\mu \frac{\mathcal{D}}{\mathcal{D}k_y} f \frac{\mathcal{D}}{\mathcal{D}k_y} n + \left(\frac{\partial \Omega}{\partial k_x}\right)^{-1} \gamma(k_y, v) n = 0 \tag{3.21}$$

which satisfies the boundary conditions (3.18). Solving the eigenvalue problem (3.18), (3.21), we find the eigenvalue $\mu(v)$ and the corresponding positive eigenfunction $\bar{n}(k_y, v)$. The point v_0 is the point at which the function $\mu(v)$ reaches its maximum. As a result of the evolution of the medium, the large-scale turbulence should go into a state in which the

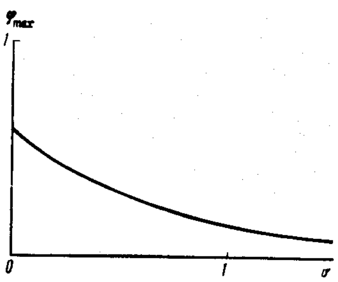

FIG. 4. The function $\varphi_{\max}(v)$. The evolution of the spectrum along the $\Omega(k) = v$ curve depends on the turbulence of the zonal flow, with wave numbers p from the sector $|p_x/p_y| < \varphi_{\max}(v)$ alone.

integral (3.19) is equal to the eigenvalue, $\mu_0 = \mu(v_0)$:

$$\int_{-\infty}^{+\infty} n(0, p_y) p_y^{u+2} dp_y = \mu_0 = \max_{0<v<+\infty} \mu(v). \tag{3.22}$$

The spectrum of the high-frequency turbulence tends toward a jet having the shape of the eigenfunction $\bar{n}(k_y, v_0)$. Its amplitude is determined from the condition that the energy flux away from the curve (which is proportional to the amplitude of the jet spectrum) be equal to the dissipation rate in the zonal flow.

If the spectrum n_p has a singularity on the k_y axis, but the spectrum of the enregy $\varepsilon_p = \omega_p n_p$ (or of some other quantity) does not have a singularity, then it is again possible to find a condition, corresponding to the condition (3.22), which fixes the level of the large-scale turbulence.

None of our arguments require that the turbulence in the zonal flow be weak. The only requirement imposed on this turbulence is that its effect on the small-scale turbulence lead to a small frequency broadening of the spectrum of the latter; specifically,

$$\delta\omega = 8\pi \int |V_{k,p,-k-p}|^2 \delta(\Omega_{k+p} - \Omega_k - \Omega_p) n_p dp \ll p_x.$$

The presence of a small-scale weak turbulence can thus fix the level of the large-scale turbulence [see (3.22)], even if it is strong.

3.6. This stabilization of large-scale turbulence by small-scale turbulence suggests yet another approach to the solution of the inverse-cascade problem.[17,18,5,39] In the theory of 2D turbulence it is assumed that energy is transferred in a step-by-step fashion from the source to large scales, at which there may be no effective dissipation, in contrast with the situation at small scales. If there is a maximum scale in the system, energy will accumulate on it (as quanta accumulate in the course of Bose condensation). In this situation it is not clear just which mechanism will operate to saturate the level of the large-scale turbulence and just which considerations are to be used to find this level. According to our results, the mechanism by which the large-scale turbulence reaches saturation might be as follows. A cascading energy flux enhances the spectrum at large scales, eventually to the point that the turbulence becomes nonlocal. The pumping of energy ceases to be a step-by-step process, and a nonlocal energy flux arises [see (3.12)–(3.14)]. The spectrum of the small-scale turbulence becomes a jet, whose amplitude is determined by the dissipation rate in the large-scale turbulence. If there is no dissipation at all at large scales, the amplitude of the jet vanishes, and the large-scale turbulence reaches saturation because it is left without a source. In this case, its level is determined unambiguously [see (3.22)]. The existence of this saturation mechanism is yet another piece of evidence in favor of the hypothesis that drift-wave turbulence is of a nonlocal nature.

3.7. Incorporating the evolutionary nonlocal nature with respect to perturbations having an odd component has led to the appearance of the second term in braces in Eq. (3.8). This term tends to make the spectrum $n(k_y, v)$ symmetric, i.e., even in k_y. Let us assume that the second term in (3.8) outweighs the first. Equation (3.8) then breaks up into systems of equations consisting of two ordinary differential equations:

$$\dot n(k_y, v)=Y(k_y, v)[n(-k_y, v)-n(k_y, v)]+\gamma(k_y, v)n(k_y, v) \quad (3.23)$$

(in each such system, k_y takes on the two values $\pm|k_y|$). It follows from (3.23) that in the limit $t\to\infty$ we have

$$\mathfrak{A}=\frac{n(k_y, v)-n(-k_y, v)}{n(k_y, v)+n(-k_y, v)}\to\frac{\theta}{1+(1+\theta^2)^{1/2}},$$

where $\theta=[\gamma(k_y,v)-\gamma(-k_y,v)]/(2Y)$. If the source is symmetric with respect to the k_x axis, the spectrum $n(k_y,v)$ becomes symmetric, i.e., even in k_y, as time elapses. If there is an asymmetric source, the relative asymmetry of the spectrum, $\mathfrak{A}$, tends toward zero with increasing intensity of the zonal flow [i.e., with increasing $Y(k_y,v)$].

By virtue of the conservation of the total number of particles on the curve, the conversion of the spectrum $n(k_y,v)$ to a symmetric form means that if the number of particles in a state with wave vector (k_x,k_y) is greater than the number of particles in the state $(k_x,-k_y)$ there will be a transition of particles from the first of these states to the second. There will be a loss of y momentum on the curve; it will be transferred to the zonal flow [see the second interval in (3.14)]. Since this transfer of y momentum is not accompanied by a transfer of energy or enstrophy [see (3.12) and (3.13)], some of the particles in the zonal flow, in the state with the wave vector $(p_x,-2k_y)$, will go into the state $(p_x,2k_y)$. There is thus a nonlocal flux of y momentum into the zonal flow, which implies that the zonal flow rotates (when the condition $n_k=n_{-k}$ is taken into account): The direction of the zonal flow acquires a component along the gradient of the inhomogeneity (the y axis). This effect might prove important in research on the anomalous-transport problem.[5,6]

3.8. Since the number of particles is conserved on each of the curves (3.9), it is natural to take up the problem of a Kolmogorov spectrum with a flux of particles along the curve. Let us assume for simplicity that the quantities $S(k_y,v)$, $\gamma(k_y,v)$, and thus the steady-state spectrum $n(k_y,v)$ are even functions of k_y. We assume that we have $\gamma(k_y,v)\equiv 0$ in some interval $a(v)<k_y<b(v)$. In the interval $a<k_y<b$ the steady-state spectrum is then found from the condition that the flux of particles along the curve remain constant:

$$Q\equiv-S\frac{\mathscr{D}n}{\mathscr{D}k_y}=\bar Q=\text{const.}$$

Hence

$$n(k_y, v)=-\bar Q\int_a^{k_y}\frac{dk_y'}{S(k_y', v)}+A,\qquad A=\text{const.}$$

In the approximation in which (3.19) does not depend on k we have

$$n(k_y, v)=-\frac{\bar Q}{\mu}\int_a^{k_y}\frac{dk_y'}{f(k_y', v)}+A.$$

This Kolmogorov solution is a two-parameter solution and also incorporates thermodynamic spectra (with a uniform distribution of particles with respect to degrees of freedom) $n=\text{const}$, $\bar Q=0$.

In summary, the jet spectrum formed as a result of the evolution of the medium is a Kolmogorov spectrum in the interval $a(v_0)<k_y<b(v_0)$.

We wish to thank L. P. Kadanoff, C. Liu, W. Horton, V. S. Shrira, A. M. Rubenchik, and K. H. Spatschek for interest in this study and for useful discussions.

APPENDIX A

Some comments on the Hamiltonian formalism for drift waves

A.1. According to the form of dispersion relation (1.1) and relation (1.3), the complex amplitudes $a_k(t)$ of waves with wave vectors k from the half-space $k_x\geqslant 0$ are independent phase variables for a system of drift waves [in real space, this system is described by the single real function $\Psi(x,y,t)$]. In this case the dynamic equation is written

$$i\dot a_k=\frac{\delta H}{\delta a_k^*},\qquad k_x>0 \quad (A.1)$$

with the Hamiltonian

$$H=\int_{k_x>0}\omega_k a_k a_k^* dk+\frac{1}{3}\sum_{s,s_1,s_2}\int_{\substack{k_x>0\\k_{1x}>0\\k_{2x}>0}}V_{k,k_1,k_2}^{ss_1s_2}\delta(sk+s_1k_1+s_2k_2)\,a_k^s a_{k_1}^{s_1} a_{k_2}^{s_2} dk_1 dk_2 dk, \quad (A.2)$$

where s, s_1, and s_2 take on the two values ±1; and $a_k^1=a_k$, $a_k^{-1}=a_k^*$. Since the dispersion relation (1.1) is of a decay type, the nonlinear interaction of the waves is incorporated in the lowest order (in the degree of the nonlinearity) by a Hamiltonian which is cubic in the wave amplitudes a_k; higher-order terms are discarded.[24] Introducing the matrix element V_{k,k_1,k_2}, which is defined for all values of the wave vectors (not solely for those with positive exponents) by

$$V_{k,k_1,k_2}^{ss_1s_2}=V_{sk,s_1k_1,s_2k_2},$$

we write Hamiltonian (A.2) in the form

$$H=\frac{1}{2}\int|\omega_k|a_k a_{-k}dk+\frac{1}{3}\int V_{k,k_1,k_2}\delta(k+k_1+k_2)a_k a_{k_1}a_{k_2}dk\,dk_1\,dk_2, \quad (A.3)$$

and we write the dynamic equation (A.1) in the form

$$i\dot a_k=\delta H/\delta a_{-k},$$

which holds for $k_x>0$. Taking the complex conjugates c both sides of this equation, and replacing k by $-k$, we find dynamic equation for $k_x<0$:

$$i\dot a_k=-\delta H/\delta a_{-k}.$$

For arbitrary k, the dynamic equation is

$$i\dot a_k=\frac{\delta H}{\delta a_{-k}}\operatorname{sign}k_x. \quad (A.4)$$

Substituting the Hamiltonian (A.3) into this equation, w find Eq. (1.2). Noting that the Hamiltonian is real and th: the second integral in (A.3) is symmetric with respect to k_1, and k_2, we find relations (1.4).

A.2. Equation (A.4) obviously conserves integrals the energy H and the momentum

$$\mathscr{P}=\frac{1}{2}\int k a_k a_{-k}\,\text{sign}\,k_x dk, \tag{A.5}$$

whose x component has an essentially positive density $|k_x|$ $|a_k|^2$ and is called the "enstrophy."

A.3. Physical equations often are not in explicit Hamiltonian form (A.4) and are instead written in the form

$$i\dot{\varphi}_k=\omega_k\varphi_k+\text{sign}\,k_x\int W_{-k,k_1,k_2}\delta(-k+k_1+k_2)\varphi_{k_1}\varphi_{k_2}dk_1\,dk_2, \tag{A.6}$$

where $\varphi_k(t)$ is the Fourier transform of the function $\Psi(x,y,t)$, which describes the state of the medium in real space and is given by

$$\Psi(x,y,t)=\frac{1}{2\pi}\int\varphi_k(t)\exp\{i(k_x x+k_y y)\}dk;$$

here

$$\varphi_k=\overset{*}{\varphi}_{-k},\quad \overset{*}{W}_{-k,-k_1,-k_2}=W_{k,k_1,k_2}=W_{k,k_2,k_1},$$

but there are no other symmetries of the type (1.4). The transformation

$$a_k=g_k\varphi_k+\text{sign}\,k_x\int G_{k,k_1,k_2}\delta(k-k_1-k_2)\varphi_{k_1}\varphi_{k_2}\,dk_1\,dk_2 \tag{A.7}$$

then makes it possible to find a dynamic equation for the new variable $a_k(t)$ which, after nonlinear terms of order higher than the second (in a_k) are discarded, has the form in (1.2), where

$$V_{k,k_1,k_2}=\frac{1}{g_{k_1}g_{k_2}}[g_{-k}W_{k,k_1,k_2}+(\omega_k+\omega_{k_1}+\omega_{k_2})G_{k,k_1,k_2}]. \tag{A.8}$$

If the resulting equation is to be a Hamilton's equation, i.e., if symmetry relations (1.4) are to hold, the function g_k must satisfy the condition $g_k=\overset{*}{g}_{-k}$, and the function $|g_k|^2W_{k,k_1,k_2}$ must have symmetries of the type in (1.4), at least on the resonance manifold:

$$k+k_1+k_2=0,\quad \omega_k+\omega_{k_1}+\omega_{k_2}=0. \tag{A.9}$$

On the one hand, these conditions determine the choice of the function g_k (clearly, this function can always be regarded as a positive, real, even function); on the other hand, they show what the function W_{k,k_1,k_2} must be if the problem is to be put in Hamilton's form. Choosing the matrix element in the form

$$V_{k,k_1,k_2}=\frac{1}{3g_k g_{k_1}g_{k_2}}[|g_k|^2W_{k,k_1,k_2}+|g_{k_1}|^2W_{k_1,k,k_2}+|g_{k_2}|^2W_{k_2,k_1,k}],$$

we find the coefficient G of the transformation (A.7) in accordance with (A.8):

$$G_{k,k_1,k_2}=[g_{k_1}g_{k_2}V_{k,k_1,k_2}-g_{-k}W_{k,k_1,k_2}]/(\omega_k+\omega_{k_1}+\omega_{k_2}).$$

The function g is usually chosen in such a way that the quadratic part of the physical energy E is equal to $\frac{1}{2}\int|\omega_k|\times a_k a_{-k}dk$ for drift waves. For example,

$$E=\frac{1}{2}\int[\Psi^2+(\rho\nabla\Psi)^2]dxdy=\frac{1}{2}\int(1+\rho^2k^2)\varphi_k\varphi_{-k}dk.$$

Hence

$$g_k=\frac{1+\rho^2k^2}{(\beta|k_x|)^{1/2}}.$$

This situation arises, for example, in the Charni-Hasegawa-Mima equation (1.5). The canonical variables $a_k(t)$ were introduced in Ref. 3; in terms of those variables, the Charney-Hasegawa-Mima equation is of the form of a Hamilton's equation in all orders (in the nonlinearity).

A.4. The kinetic equation for waves corresponding to the dynamic equation (A.1) is[24]

$$\frac{\partial n}{\partial t}=St[n]=\int_{k_{1x},k_{2x}>0}(R_{012}-R_{102}-R_{210})dk_1dk_2,$$

where

$$R_{012}=4\pi|V^{-1,1,1}_{k,k_1,k_2}|^2\delta(k-k_1-k_2)\delta(\omega_k-\omega_{k_1}-\omega_{k_2})$$
$$\times(n_{k_1}n_{k_2}-n_kn_{k_1}-n_kn_{k_2}).$$

This equation becomes the kinetic equation (2.6) if we switch from integration over the half-spaces $k_{1x}>0$ and $k_{2x}>0$ in the collision integral to integration over all wave vectors, making use of the condition $n_k=n_{-k}$ and symmetry relations (1.4).

APPENDIX B

Nonlocal turbulence in isotropic media

B.1. For comparison, we consider nonlocal turbulence in an isotropic medium with a dispersion relation $\omega=|k|^\alpha$. If $\alpha>1$, this dispersion relation is of a decay type, and the kinetic equation for waves is[24]

$$\dot{n}_k=\int(R_{k,k_1,k_2}-R_{k_1,k,k_2}-R_{k_2,k_1,k})dk_1dk_2+\gamma_kn_k, \tag{B.1}$$

where

$$R_{k,k_1,k_2}=4\pi|V_{k,k_1,k_2}|^2\delta(k-k_1-k_2)\delta(\omega_k-\omega_{k_1}-\omega_{k_2})$$
$$\times(n_{k_1}n_{k_2}-n_kn_{k_1}-n_kn_{k_2}).$$

If the interaction with large scales is the dominant interaction, we can restrict (B.1) to an integration over small $|k_1|$ (and take small $|k_2|$ into account by doubling the result). To first order (in the ratio of the small and large scales, which is a small parameter), we can ignore the quantity ω_{k_1} in Eq. (B.1), since it is small in comparison with the difference $\omega_k-\omega_{k_2}\approx(\partial\omega_k/\partial k)k_1$. In lowest order, the evolution of the spectrum n_k in k space thus occurs independently on each of the circles

$$\omega_k=\omega=\text{const},\quad |k|=\text{const}. \tag{B.2}$$

The high-frequency quanta move in a manner which conserves their energy ω_k against the background of the "frozen" large-scale turbulence. In the case of nonlocal drift-wave turbulence, we ignored the quantity Ω_{k1}, assuming that the large-scale turbulence is frozen in the coordinate system moving at the drift velocity β. In the lowest order, the total number of particles on the curve in (B.2),

$$N_\omega=\int n_kd\varphi$$

(φ is the polar angle of the vector k), obeys the equation

$$\dot{N}_\omega=\int\gamma_kn_kd\varphi. \tag{B.3}$$

In isotropic media, on some of the circles (B.2), the quantity γ_k is usually positive everywhere, so the number of particles $N(\omega)$ on these circles will increase without bound according to (B.3). It is then no longer sufficient to retain only the lowest order in the nonlocal interaction; we need to incorporate higher orders, which describe a redistribution of particles among circles. The validity of using only the first approximation in the case of a drift-wave turbulence is based in a fundamental way on the circumstance that any curve of the type (3.9) passes through a region of strong dissipation.

B.2. If γ_k is isotropic $[\gamma=\gamma(|k|)]$, the interactions among circles (B.2) then lead to a rapid conversion of the spectrum n_k to an isotropic form. Then there is a slower redistribution of particles among the circles, which is described by the average kinetic equation[21] (averaged over angle)

$$\omega^{2/\alpha-1}\dot{N}_\omega=\int(R_{\omega,\omega_1,\omega_2}-R_{\omega_1,\omega,\omega_2}-R_{\omega_2,\omega_1,\omega})\,d\omega_1 d\omega_2+\gamma_\omega N_\omega,$$
$$R_{\omega,\omega_1,\omega_2}=U_{\omega,\omega_1,\omega_2}\delta(\omega-\omega_1-\omega_2)\,[N_{\omega_1}N_{\omega_2}-N_\omega N_{\omega_1}-N_\omega N_{\omega_2}], \quad \text{(B.4)}$$

where $U_{\omega,\omega_1,\omega_2}$ is a function determined by the matrix element of the medium, which satisfies $U_{\omega,\omega_1,\omega_2}=U_{\omega,\omega_2,\omega_1}$. If the medium is scale-invariant, the function $U_{\omega,\omega_1,\omega_2}$ is a homogeneous function of some power of λ:

$$U(\varepsilon\omega,\varepsilon\omega_1,\varepsilon\omega_2)=\varepsilon^\lambda U(\omega,\omega_1,\omega_2).$$

In this case we have the Kolmogorov spectrum

$$N_\omega=CP^{1/2}\omega^{-\nu},\quad \nu=(\lambda+3)/2, \quad \text{(B.5)}$$

where P is the energy flux to small scales, and C is a dimensionless constant. This spectrum is an exact solution of Eqs. (B.4), (B.2) under the condition that it is steady and local (i.e., under the condition that the collision integral converges for this spectrum). If the Kolmogorov spectrum (B.5) is nonlocal in an evolutionary sense with respect to anisotropic perturbations alone, the nonlocal nature of the turbulence is manifested only in the stage in which the spectrum becomes isotropic. The subsequent evolution of the spectrum is determined by the interaction between nearby scales. The spectrum (B.5) may be established in the process.

If the Kolmogorov spectrum (B.5) is also nonlocal with respect to isotropic perturbations, then the nonlocal interaction with large scales determines the evolution of even isotropic spectra. Equation (B.4) reduces to the differential equation

$$\omega^{2/\alpha-1}\dot{N}_\omega=-\frac{\partial}{\partial\omega}S\frac{\partial}{\partial\omega}N_\omega+\gamma_\omega N_\omega,$$

where

$$S=2\int U_{\omega,\omega_1,\omega-\omega_1}\omega_1^2 N\omega_1 d\omega_1. \quad \text{(B.6)}$$

If the asymptotic formula

$$U_{1,\omega_1,1-\omega_1}=u\omega_1^q,\quad u=\text{const},$$

holds in the limit $\omega_1\to 0$, then we have $S=S_0\omega^{\lambda-q}$, where $S_0=2u\int_0^\infty N_{\omega_1}\omega_1^{q+2}d\omega_1$, and Eq. (B.6) becomes

$$\omega^{2/\alpha-1}\dot{N}_\omega=-S_0\frac{\partial}{\partial\omega}\omega^{\lambda-q}\frac{\partial}{\partial\omega}N_\omega+\gamma_\omega N_\omega, \quad \text{(B.7)}$$

since the effect of the large-scale turbulence on the small-scale turbulence is described by the single constant S_0. In the case $\gamma=0$, this equation conserves the total number of particles, and the general steady-state solution of this equation is the sum of a Kolmogorov spectrum (with a particle flux Q) and a thermodynamic-equilibrium spectrum (with a uniform distribution of particles among degrees of freedom):

$$N=\frac{Q}{S_0}\omega^{-\tilde{\nu}}+\text{const},\quad \tilde{\nu}=\lambda-q-1.$$

[1] Finite perturbations correspond to the case in which the primary source and the primary sink which shape the Kolmogorov spectrum, and which are concentrated at the edges of the inertial interval, remain constant. Such perturbations are always present in a medium because of internal or external fluctuations of the system.

[2] Finite perturbations form a natural class of initial conditions in the formulation of a Cauchy problem for the linearized kinetic equation.[30]

[3] No distinction was made in those studies between nonlocalness which is a consequence of the divergence of a collision integral for a finite perturbation of the spectrum and nonlocalness which is a consequence of a divergence for a spectrum which arises as a result of the evolution of a finite initial perturbation. The two cases were lumped together in the concept of evolutionary nonlocalness which was introduced in Refs. 28–32 (in this case, the evolution of perturbations is not determined exclusively by near-scale interactions).

[4] Strictly speaking, what is scale-invariant in the case of dispersion relation (2.14) is not the frequency ω_k but the quantity $\beta k_x-\omega_k$, which may be interpreted as the frequency in a coordinate system moving at a velocity β. In this case, P in expression (2.11) represents not the energy flux $\mathscr{E}$ but the energy flux measured in the coordinate system moving at a velocity β: $\mathscr{E}'=\beta\mathscr{P}_x-\mathscr{E}$. For simplicity, however, we will not be specifying just which energy we have in mind.

[1] M. S. Longuet-Higgins and A. E. Gill, *Nonlinear Theory of Wave Propagation* (Russ. Transl. Mir, Moscow, 1970, p. 161).
[2] A. Hasegawa and K. Mima, Phys. Fluids **21**, 87 (1978).
[3] V. E. Zakharov and L. I. Piterbarg, Dokl. Akad. Nauk SSSR **295**, 86 (1987) [Sov. Phys. Dokl. **32**, 560 (1987)].
[4] V. M. Kamenkovich, M. N. Koshlyakov, and A. S. Monin, *Synoptic Eddies in Ocean*, Spring-Verlag, New York, 1985.
[5] A. Hasegawa, C. G. Maclennan, and Y. Kodama, Phys. Fluids **22**, 212 (1979).
[6] W. Horton, in *Basic Plasma Physics, Vol. 2* (eds. A. A. Galeev and R Sudan), North-Holland, Amsterdam, 1984 (Russ. Transl. Énergoato mizdat, Moscow, 1984).
[7] J. F. Drake, P. N. Guzdar, A. B. Hassam, and J. D. Huba, Phys. Fluid **27**, 1148 (1984).
[8] A. S. Kingsep, K. V. Chukbar, and V. V. Yan'kov, in *Reviews of Plasma Physics, Vol. 16* (ed. B. B. Kadomtsev), Consultants Bureau, New York 1989.
[9] B. B. Kadomtsev and O. P. Pogutse, in *Reviews of Plasma Physics, Vol.* . (ed. M. A. Leontovich), Consultants Bureau, New York (1970).
[10] A. V. Mikhailovskii, S. V. Nazarenko, A. P. Churikov, and O. G. Onish chenko, Plasma Phys. (To be published).
[11] V. V. Dolotin and A. M. Fridman, *Dynamics of Astrophysical Discs* (ed J. Seilwood), Cambridge Univ. Press, Oxford, 1989, p. 71.
[12] H. Tasso, Phys. Lett. A **24**, 618 (1967).
[13] A. B. Mikhailovskii, V. P. Lakhin, E. A. Novakovskaya, and O. G Onishchenko, Phys. Lett. A **132**, 39 (1988).
[14] L. F. Richardson, *Weather Prediction by Numerical Process*, Cambridg Univ. Press, Oxford, 1922.
[15] A. N. Kolomogorov, Dokl. Akad. Nauk SSSR **30**, 299 (1941).
[16] A. M. Obukhov, Izv. Akad. Nauk SSSR Geogr. i geofiz. **5** 453 (1941).
[17] R. H. Kraichnan, Phys. Fluids **10**, 1417 (1967).
[18] R. H. Kraichnan and D. Montgomery, Rep. Progr. Phys. **43**, 54 (1980).
[19] V. E. Zakharov, Zh. Prikl. Mekh. Tekh. Fiz. No. **4**, 35 (1965).
[20] V. E. Zakharov, Zh. Eksp. Teor. Fiz. **51**, 688 (1966) [Sov. Phys. JET **24**, 455 (1967)].
[21] V. E. Zakharov, in *Basic Plasma Physics, Vol.2* (eds. A. A. Galeev ar R. Sudan), North-Holland, Amsterdam, 1984.
[22] B. B. Kadomtsev, in *Reviews of Plasma Physics, Vol. 4* (ed. M. A. Leo tovich), Consultants Bureau, New York (1966).
[23] A. A. Galeev and R. Z. Sagdeev, in *Reviews of Plasma Physics, Vol.* (ed. M. A. Leontovich), Consultants Bureau, New York (1978).
[24] V. E. Zakharov, Izv. Vyssh. Uchebn. Zaved., Radiofiz. **17**, 431 (1974
[25] E. A. Kuznetsov, Zh. Eksp. Teor. Fiz. **62**, 584 (1972) [Sov. Phys. JET **35**, 310 (1972).
[26] A. S. Monin and L. I. Piterbarg, Dokl. Akad. Nauk SSSR **295**, 8 (1987) [Sov. Phys. Dokl. **32**, 622 (1987)].
[27] A. B. Mikhaĭlovskiĭ, S. V. Novakovskiĭ, V. P. Lakhin *et al.*, Zh. Eks Teor. Fiz. **94**(7), 159 (1988) [Sov. Phys. JETP **67**, 1386 (1988)]

[28]A. M. Balk and V. E. Zakharov, Dokl. Akad. Nauk SSSR **299**, 1112 (1988) [Sov. Phys. Dokl. **33**, 270 (1988).
[29]A. M. Balk and V. E. Zakharov, *Plasma Theory and Nonlinear and Turbulent Processes in Physics. Proceedings of International Workshop*, April 1987, Kiev, World Scientific, Singapore, 1988, p. 359.
[30]A. M. Balk and V. E. Zakharov, *Integrability and Kinetic Equations for Solitons* (eds. V. G. Bar'yakhtar, V. E. Zakharov, and V. M. Chernousenko), Naukova dumka, 1990 (in press).
[31]A. M. Balk, S. V. Nazarenko, and V. E. Zakharov, in *Proceedings of the Fourth International Workshop on Nonlinear and Turbulent Processes in Physics, Vol. 2*, 1989, p. 342.
[32]A. M. Balk, S. V. Nazarenko, Zh. Eksp. Teor. Fiz. **97**, 1827 (1990) [Sov. Phys. JETP
[33]A. B. Mikhailovskii, S. V. Nazarenko, S. V. Novakovskii *et al.*, Phys. Lett. A **133**, 407 (1988).
[34]M. V. Nezlin, Usp. Fiz. Nauk **150**, 3 (1986) [Sov. Phys. Usp. **29**, 807 (1986)].
[35]J. Labelle, M. C. Kelley, C. E. Seylor *et al.*, J. Geophys. Res. **91**, 5513 (1986).
[36]M. Kono and E. Miyashita, Phys. Fluids **31**, 326 (1988).
[37]W. Horton, Phys. Fluids **29**, 1491 (1986).
[38]A. A. Vedenov and L. I. Rudakov, Dokl. Akad. Nauk SSSR **159**, 767 (1964) [Sov. Phys. Dokl. **9**, 1073 (1965)].
[39]D. Fyfe and D. Montgomery, Phys. Fluids **22**, 246 (1979).

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