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Breakdown of wave turbulence and the onset of intermittency

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Abstract

This Letter demonstrates that the kinetic equations for wave turbulence, the long time statistical behavior of a sea of weakly coupled, dispersive waves, will almost always develop solutions for which the theory fails due to strongly nonlinear and intermittent events either at small or large scales. © 2001 Elsevier Science B.V. All rights reserved.

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Intermittency connotes burstiness and intermittent signals display off-again, on-again characteristics in which large fluctuations have significant impact on higher order statistical moments. In three-dimensional, hydrodynamic turbulence at high Reynolds numbers, it is measured by studying the probability density function (pdf) of velocity differences $\delta v(\vec{x},\vec{r}) = v(\vec{x}+\vec{r}) - v(\vec{x})$ and their higher order moments, the structure functions $S_N(\vec{r}) = \langle (v(\vec{x}+\vec{r}) - v(\vec{x}))^N \rangle$, $N \geqslant 2$, as functions of separation distance r. For small values of r at the high wavenumber end of the inertial range, but larger than the dissipation scale, it is found that the tails of the pdf for $\delta v(\vec{x},\vec{r})$ increase as r decreases. Large fluctuations dominate the higher order moments [1]. As a result, the ratio of S_N to $(S_2)^{N/2}$ which, from Kolmogorov '41 theory [2], in the long time, infinite Reynolds number limit would be constant, diverges as $r \to 0$. The events causing intermittent behavior are believed to be associated with large fluctuations in the local dissipation rate but are not well understood. In contrast, the long time statistical behavior of the one-dimensional turbulence associated with Burgers' equation exhibits similar behavior which is well understood and is associated with the formation and accretion of shocks [3].

The surprising new result of this Letter (and it is surprising considering that kinetic equations have been studied in various contexts for well over sixty years [4–6]!) is that, in non-equilibrium situations, a wave turbulence field fed by sources and drained by sinks, well separated in wavenumber space, develops solutions so that, at a well defined length scale k_{NL}^{-1}, the weak coupling approximation fails. The system departs from joint-Gaussian statistics and strongly nonlinear and intermittent events affect the dynamics. We illustrate the breakdown in the contexts of deep water gravity waves and optical waves in nonlinear media.

We begin by summarizing wave turbulence theory, the reasons for asymptotic closure, the Kolmogorov–Zakharov (KZ) finite flux solutions and then present the reasons for its breakdown. Let $v(\vec{x},t)$ be a spatially

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homogeneous, random, zero mean function of position $\vec{x}$, the Fourier transform of whose pair correlation function $\langle v(\vec{x},t)v(\vec{x}+\vec{r},t)\rangle$ is the spectral energy density $e(\vec{k})$. For deep water gravity waves, $v(\vec{x},t)$ is the surface deformation $\eta(\vec{x},t)$. We decompose $v(\vec{x},t)$ into the normal modes $v^s(\vec{x},t)$ (left and right going waves $\exp(i\vec{k}\cdot\vec{x}+is\omega(\vec{k})t)$, $s=+,-$) of its linear approximation ($\omega(\vec{k})\geqslant 0$, the linear dispersion relation, e.g., $\omega=\sqrt{gk}$, $k=|\vec{k}|$) and let $\sqrt{\omega}A^s(\vec{k},t)$, $s=+,-$, be their (generalized) Fourier transforms. To leading order in amplitude, the spectral energy $e(\vec{k})$ is $(1/2)\sum_s \omega(\vec{k})n^s(\vec{k})$, where, using spatial homogeneity, $\langle A^s(k,t)A^{-s}(k',t)\rangle=\delta(\vec{k}+\vec{k}')n^{-s}(\vec{k},t)$. For simplicity, we take $n^+=n^-=n(k,t)=n_k$ and then $e(\vec{k})=\omega(k)n_k$, where n_k is called the wave action or particle number. It is natural to write the field equation in Fourier coordinates,

$$\begin{aligned}
&\frac{dA^s(\vec{k})}{dt}-is\omega(\vec{k})A^s(\vec{k})\\
&\quad=\sum_{s_1s_2}\int L^{ss_1s_2}_{\vec{k}\vec{k}_1\vec{k}_2}A^{s_1}(\vec{k}_1)A^{s_2}(\vec{k}_2)\\
&\qquad\times\delta(\vec{k}_1+\vec{k}_2-\vec{k})\,d\vec{k}_1\,d\vec{k}_2\\
&\qquad+\sum_{s_1s_2s_3}\int L^{ss_1s_2s_3}_{\vec{k}\vec{k}_1\vec{k}_2\vec{k}_3}A^{s_1}(\vec{k}_1)A^{s_2}(\vec{k}_2)A^{s_3}(\vec{k}_3)\\
&\qquad\times\delta(\vec{k}_1+\vec{k}_2-\vec{k}_3-\vec{k})\\
&\qquad\times d\vec{k}_1\,d\vec{k}_2\,d\vec{k}_3+\cdots.
\end{aligned}\tag{1}$$

Since most wave turbulence systems exchange spectral energy via three or four wave resonances, it is sufficient to retain only quadratic and cubic terms in (1), the most general form of Hamilton's equations for waves in a dispersive medium; e.g., gravity, surface tension, optical and Alfven waves. Wave turbulence theory begins by writing the hierarchy of equations for the spectral cumulants

$$\begin{aligned}
&Q^{(N)ss'\ldots s^{N-1}}(\vec{k}',\vec{k}'',\ldots,\vec{k}^{(N-1)},t)\\
&\quad=(2\pi)^{-(N-1)d}\\
&\qquad\times\int R^{(N)ss'\ldots s^{N-1}}(\vec{r},\vec{r}',\ldots,\vec{r}^{(N-2)},t)\\
&\qquad\times\exp(-i\vec{k}'\cdot\vec{r}-\cdots-i\vec{k}^{N-1}\cdot\vec{r}^{N-2})\\
&\qquad\times d\vec{r}\ldots d\vec{r}^{N-2},
\end{aligned}$$

where $R^{(2)ss'}(\vec{r})=\langle v^s(x)v^{s'}(\vec{x}+\vec{r})\rangle$ and $R^{(N)}$ is the spatial cumulant of order N obtained by subtracting the unique combination of products of moments from the Nth order moment so that $R^{(N)}$ decays as $|\vec{r}|,\ldots,|\vec{r}^{N-1}|$ tend independently to infinity. The hierarchy is solved iteratively by writing each $Q^{(N)}$ in a power series in amplitude (ε say, $0<\varepsilon\ll 1$) and evaluating the behavior of each iterate (or, where appropriate, its physical space transform) in the limit $\omega_0 t\to\infty$, $\varepsilon^r\omega_0 t$ fixed for $r=2,4,\ldots,\omega_0$ some typical frequency. Non-uniformities in time in the asymptotic expansion are removed by choosing the ε^2, ε^4 coefficients in an asymptotic expansion for

$$\frac{dn^s_{\vec{k}}}{dt}=T_2[n^s_{\vec{k}}]+T_4[n^s_{\vec{k}}]+\cdots\tag{2}$$

and by renormalizing the frequency

$$\begin{aligned}
&s\omega_k\to s\omega_k+s\Omega^s_2[n^s_k]+\cdots,\\
&\Omega^s_2[n^s_k]=\int G_{\vec{k}\vec{k}_1-\vec{k}_1\vec{k}}n_{\vec{k}_1}\,d\vec{k}_1.
\end{aligned}\tag{3}$$

(The function G is defined below.) Closure is achieved for two reasons. First, on time scales long with respect to ω_0^{-1} but short with respect to $(\varepsilon^2\omega_0)^{-1}$, the statistics approach a state close to joint Gaussian because of the dispersive nature of the waves. Second, the regeneration of the cumulant $Q^{(N)}$, $N\geqslant 3$, by nonlinear terms is dominated over long times not by $Q^{(M)}$, $M>N$, but by products of cumulants of order less than or equal to N. In particular, Eq. (2) for the wave action density $n^s_{\vec{k}}$ (and thereby $e(\vec{k})$) is closed. It is called the kinetic equation. $T_2[n^s_{\vec{k}}]$ is equal to

$$\begin{aligned}
&4\pi\sum_{s_1s_2}\int\left|L^{ss_1s_2}_{kk_1k_2}\right|^2 n^s_{\vec{k}}n^{s_1}_{\vec{k}_1}n^{s_2}_{\vec{k}_2}\left(\frac{s}{n^s_{\vec{k}}}-\frac{s_1}{n^{s_1}_{\vec{k}_1}}-\frac{s_2}{n^{s_2}_{\vec{k}_2}}\right)\\
&\qquad\times\delta(s_1\omega_1+s_2\omega_2-s\omega)\\
&\qquad\times\delta(\vec{k}_1+\vec{k}_2-\vec{k})\,d\vec{k}_1\,d\vec{k}_2
\end{aligned}$$

and is nontrivial when the *three wave resonant manifold* defined by $s_1\omega(\vec{k}_1)+s_2\omega(\vec{k}_2)=s\omega(\vec{k}_1+\vec{k}_2)$ is nonempty for some choices of s,s_1,s_2. It redistributes energy density on the time scale $(\varepsilon^2\omega_0)^{-1}$. If the three wave resonant manifold is empty (which it is, for example, if $\omega=ck^\alpha$, $\alpha<1$), $T_2=0$, then four wave resonances redistribute energy density on the time scale $(\varepsilon^4\omega_0)^{-1}$. Assuming $n^+=n^-$ and summing over s_j, the collision integral $T_4[n_k]$ is

$$\sum_{s_1s_2s_3}\int \left|G_{\vec{k}\vec{k}_1\vec{k}_2\vec{k}_3}\right|^2 n_{\vec{k}}n_{\vec{k}_1}n_{\vec{k}_2}n_{\vec{k}_3} \times\left(\frac{1}{n_{\vec{k}}}+\frac{1}{n_{\vec{k}_1}}-\frac{1}{n_{\vec{k}_2}}-\frac{1}{n_{\vec{k}_3}}\right) \times\delta(\omega+\omega_1-\omega_2-\omega_3) \times\delta\big(\vec{k}+\vec{k}_1-\vec{k}_2-\vec{k}_3\big)\,d\vec{k}_1\,d\vec{k}_2\,d\vec{k}_3,$$

where $G_{\vec{k}\vec{k}_1\vec{k}_2\vec{k}_3}$ is a linear combination of $L^{ss_1s_2s_3}_{\vec{k}\vec{k}_1\vec{k}_2\vec{k}_3}$ and a quadratic product of $L^{ss_1s_2}_{\vec{k}\vec{k}_1\vec{k}_2}$ divided by frequency. For three wave processes, one has (formally) conservation of energy $\int e(\vec{k})\,d\vec{k}$; for four wave processes, both energy and total wave action are formally conserved. The Rayleigh–Jeans solution of energy density equipartition $n^s_{\vec{k}} = T\omega_{\vec{k}}^{-1}$, and, in the case of four wave processes, $n_k = T(\omega_k-\mu)^{-1}$ are obvious.

Finite flux solutions of (2), $n_k = n_k(\tau, P)$ for three wave processes, $n_k = n_k(\tau,\mu,P,Q)$ for four wave processes, where the parameters τ,μ,P,Q, correspond to temperature, chemical potential, energy and wave action fluxes respectively, are less obvious. However, in cases for which the dispersion ω_k and coupling coefficients $L^{ss_1s_2}_{k,k_1,k_2}, L^{ss_1s_2s_3}_{k,k_1,k_2,k_3}$ are homogeneous in k of degrees α,β,γ, respectively, power law (KZ) solutions can be found by comparing powers of k in

$$\frac{\partial E(k)}{\partial t} = -\frac{\partial P}{\partial k},\quad E(k)=\Omega\omega_k k^{d-1}n_k$$

(Ω is the solid angle in d dimensions) and in

$$\frac{\partial}{\partial t}N(k)=\frac{\partial Q}{\partial k},\quad N(k)=\Omega k^{d-1}n_k.$$

These cases are not rare [5,6]. For water waves, ω_k and the coupling coefficients are homogeneous for roughly four decades of wavelength. For three wave processes $n_k = c_1P^{1/2}k^{-(\beta+d)}$ (P is of order ε^4) describes the constant flux of energy P from a source at low k to a sink at high wavenumbers, a forward cascade. For four wave processes, $n_k = c_2P^{1/3}k^{-(2\gamma/3+d)}$ describes the forward cascade of energy while $n_k = c_3Q^{1/3}k^{-(2\gamma/3+d)+\alpha/3}$ describes the inverse/back cascade of wave action from a source at intermediate (k_1) wavenumber to a sink at low wavenumber. (P,Q are of order ε^6.) Observe that $\int_{k_1}^{\infty}E(k)\,dk$ converges for $\beta>\alpha$ for three wave processes and for $(2\gamma>3\alpha)$ for four wave processes where also $\int_0^{k_1}N(k)\,dk$ converges for $\alpha>2\gamma$. We then say that the spectrum has *finite* rather than infinite capacity in that it can only absorb a finite amount of energy (wave action) and requires a finite time to establish the equilibrium spectrum and make a connection to a sink.

For almost all choices of α,β,γ, the premises on which wave turbulence theory is based are violated. An explicit formula for the length scale at which breakdown occurs is given in terms of α,β,γ,P,Q. While here we carry out explicit calculations for cases in which the KZ power spectra obtain, we emphasize the phenomenon of breakdown is general and a consequence of all finite flux solutions which carry energy and/or wave action to scales at which the weak coupling approximation may fail. In order to calculate this scale we examine the (i) ratio of linear, t_L, to nonlinear, t_{NL}, times where

$$t_L \propto \omega_{\vec{k}}^{-1}\quad\text{and}\quad t_{NL}\propto\frac{1}{n_k}\frac{dn_k}{dt}=\frac{1}{n_k}T_2;$$

(ii) ratios of terms in the ε-expansion of the kinetic equation T_{2N+2}/T_{2N}, $N=1,2,\ldots$; (iii) deviations from joint-Gaussianity as given by the ratio of structure functions $S_N(\vec{r})$ to $S_2^{(N/2)}(\vec{r})$ for small r and the ratios of the cumulants $R_N(\vec{r})$ to $R_N^{(N/2)}(\vec{r})$ for large r. All three criteria give the *same* breakdown scale. On the KZ energy flux spectrum, the ratio t_L/t_{NL} is $P^{1/2}k^{\beta-2\alpha}$ for three wave interactions and $P^{2/3}k^{2\gamma/3-2\alpha}$ for four wave; the wave action spectrum gives $Q^{2/3}k^{2\gamma/3-4\alpha/3}$. To see this, note

$$\frac{t_L}{t_{NL}}=\frac{1}{\omega_k n_k}\frac{dn_k}{dt}=\frac{T_2}{\omega_k n_k}\cong\frac{k^{2\beta}Pk^{-2(\beta+d)}k^{-\alpha}k^{2d}}{k^{\alpha}P^{1/2}k^{-(\beta+d)}} = P^{1/2}k^{\beta-2\alpha}.$$

Below, we show that large deviations from joint-Gaussianity for the direct energy cascade occur at small r when $P^{1/2}r^{2\alpha-\beta}\sim 1$ for three-wave or $P^{2/3}r^{2\alpha-2\gamma/3}\sim 1$ for four-wave interactions. For the (inverse) cascade of wave action the deviations occur at large r when $Q^{2/3}r^{4\alpha/3-2\gamma/3}\sim 1$. Satisfying these relations defines a length scale, $k_{NL}^{-1}=r_{NL}$. It can happen that k_{NL} lies within the window of transparency between the forcing (k_f) and dissipation (k_d) scales in which the KZ solutions obtain. In the limit of zero forcing, $P\to 0$ and k_{NL} lies outside the window, but even for modest forcing and small k_f and large k_d, $k_f<k_{NL}<k_d$. For $\beta>2\alpha$ ($\gamma>3\alpha$), k_{NL} is large. The KZ spectrum is valid for $k_f < k < k_{NL}$ but, for $k_{NL} < k < k_d$, there is a region of anomalous scaling and complete breakdown of wave turbulence. In exactly the same range, the asymptotic series for frequency renormalization also becomes nonuniform. For $\beta < 2\alpha$ ($\gamma < 3\alpha$), k_{NL} is small and wave turbulence breaks down for $k_f < k < k_{NL}$ but is repaired in $k_{NL} < k < k_d$. On the wave action flux spectrum, k_{NL} is defined by $Q^{1/3}k_{NL}^{\gamma/3-2\alpha/3} \sim 1$ and breakdown occurs for $0 < k < k_{NL}$ when $2\alpha > \gamma$ and for $k_{NL} < k < k_f$ when $\gamma > 2\alpha$.

We now calculate (details are in [6]) for three wave interactions the Nth order structure function $S_N(\vec{r})$ (for small r; we examine R_N for large r) and the parameter ranges for which it is dominated by the universal KZ spectrum. For each expansion of a Fourier space cumulant, there is a leading term in the limit of long time which depends only on $n(\vec{k}, t)$ and no other cumulant. The first surviving part of $Q^{(3)}$ is given by

$$2P_{0,0',0''}\sqrt{\omega_k\omega_{k'}\omega_{k''}}\, L^{s-s'-s''}_{\vec{k}-\vec{k}'-\vec{k}''} n_{\vec{k}'} n_{\vec{k}''} \times \left(\pi\delta(s\omega + s'\omega' + s''\omega'') + iP(s\omega + s'\omega' + s''\omega'')^{-1}\right)$$

and determines the scaling of $S_3(\vec{r})$ as well as the integrand of T_2 in the kinetic equation (2). (Here, $\vec{k} + \vec{k}' + \vec{k}'' = 0$, $P_{0,0',0''}$ is the permutation and sum over $(s, \vec{k})$, $(s', \vec{k}')(s'', \vec{k}'')$ and δ and P are the Dirac delta function and Cauchy principal value, respectively.) Structure functions are calculated by taking the inverse Fourier transform of these surviving terms. For $N = 2$,

$$S_2(\vec{r}) = 2\left(\langle v(x)\rangle^2 - \langle v(x)v(x+r)\rangle\right) = 2\int \omega_k n_k\left(1 - \cos(\vec{k}\cdot\vec{r})\, d\vec{k}\right).$$

For $0 < \beta - \alpha < 2$, $S_2(\vec{r}) \sim P^{1/2}r^{\beta-\alpha}$. The lower limit corresponds to finite capacity. The upper limit means that the contribution of the universal part of the spectrum dominates that of the nonuniversal part which is proportional to $r^2 \ll r^{\beta-\alpha}$ for small r. Similarly we find $S_3(\vec{r}) \sim Pr^{\beta-\alpha/2}$. Likewise, the surviving part of the fourth order cumulant gives rise to the integrand in T_4, and in general, the surviving part of the Nth order cumulant $Q^{(N)}$ will give rise to the integrand in T_{2N-4}. The ratio of $S_N(\vec{r})$ to $(S_2(\vec{r}))^{N/2}$ is proportional to a finite series

$$1 + \sum_{s=1}^{N/2-1} C_{Ns}\rho^s$$

for even N and

$$\rho^{1/2}\left[1 + \sum_{s=1}^{(N-3)/2} C_{Ns}\rho^s\right]$$

for odd N where $\rho = P^{1/2}r^{2\alpha-\beta}$. One can prove that for $0 < \beta - \alpha < 2$, $0 < \alpha \leqslant 2$, the small r behavior of S_N for all N is dominated by the contributions from the KZ spectrum and not from the nonuniversal part. In calculating S_N, we use the fact that the first surviving part of the Nth order cumulant $Q^{(N)}(\vec{k})$ begins with a term proportional to the product of frequency to the $N/2$ power, the coefficient $L^{ss_1s_2}_{\vec{k}\vec{k}_1\vec{k}_2}$ to the $(N-2)$th power, the particle density n_k to the $(N-1)$th power divided by the $(N-2)$th power of sums and differences of frequencies. It should be noted that, just as in the expansion for the kinetic equation, the ratio of successive surviving terms in the cumulant expansion also give powers of $P^{1/2}r^{2\alpha-\beta}$.

Similar results hold for the energy flux spectrum for the four wave interactions when $0 < 2\gamma/3 - \alpha < 2$, $0 < \alpha \leqslant 2$. For breakdown at small k (large r), the relevant quantities are the ratios of the cumulants corresponding to the moment $\langle v^{N-1}(\vec{x})v(\vec{x}+\vec{r})\rangle$. The manifestation of breakdown in this case is divergence as $r \to \infty$ and usually corresponds to the formation of condensates. In this context, it should be mentioned that for four wave interactions and small negative $\gamma - 3\alpha$, Gurarie [7] has used a renormalization technique to suggest a corrected spectrum in $k_f < k < k_{NL}$.

One advantage of nonuniformly valid wave turbulence systems is that the nonlinear coherent structures, which are initiated at breakdown, can often be identified. For optical waves in nonlinear media described by nonlinear Schrödinger-like field equations [6,8] (a picture also relevant for Bose–Einstein condensation and the carrier distribution in semiconductor lasers), the inverse cascade of wave action to low k leads to the formation of condensates and to either a new class of fluctuations if the medium is defocusing or to collapsing filaments if it is focusing. In the latter case, the filaments are extremely coherent but are formed randomly in space-time. The parameters of their statistical distribution (uniform in space, Poisson in time) are determined by the wave action flux rate Q. The signal for the particle dissipation rate is highly intermittent with spikes (large fluctuations) directly identifiable with collapse events [8].

For deep water gravity waves ($\alpha = 1/2$, $\beta = 7/4$, $\gamma = 2\beta - \alpha = 3$), the condition for the breakdown of the KZ spectrum $E(k) \sim P^{1/3}k^{-7/2}g^{1/2}$ is that $P^{2/3}k/g$ becomes of order one for $k < k_0$, $k_0 = \sqrt{\rho_w g/S}$, the scale at which surface tension (S) is important. Here $P^{2/3} = (\rho_a/\rho_w)V^2$, where ρ_a, ρ_w are the densities of air and water respectively and V is wind speed. The criterion for breakdown turns out to be exactly the same [9] as that of the KZ spectrum intersecting the Phillips spectrum $E(k) \sim k^{-4}$ before k_0. The Phillips spectrum coincides with that of an ocean surface dominated by derivative discontinuities (sharp crests). When $V > 6$ m/s, $P^{2/3}k_0/g > 1$, suggesting that the breakdown of the KZ spectrum leads to singular solutions associated with crest formation.

Finally, we mention that the numerical evidence presented in [10] for the non-appearance of the KZ spectrum seems to be connected with a dominance of nonlocal interactions and therefore nonuniversal behavior.

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References

[1] F. Anselmet, Y. Gagne, E.J. Hopfinger, R.A. Antonio, J. Fluid Mech. 140 (1984) 63;
R. Benzi, S. Ciliberto, C. Baudet, C.R. Chavarria, Physica D 80 (1993) 385.
[2] U. Frisch, Turbulence, Cambridge University Press, 1995.
[3] W. E, K. Khanin, A. Mazel, Ya.G. Sinai, Phys. Rev. Lett. 78 (1997) 1904;
R. Tribe, O. Zaboronski, Comm. Math. Phys., to appear.
[4] R. Peierls, Quantum Theory of Solids, Clarendon, 1955;
K. Hasselmann, J. Fluid Mech. 12 (1962) 481;
B.B. Kadomtsev, Plasma Turbulence, Academic Press, 1965.
[5] V.E. Zakharov, V.S. L'vov, G. Falkovich, Kolmogorov Spectra of Turbulence, Vol. 1, Springer, 1992.
[6] A.C. Newell, S.V. Nazarenko, L. Biven, Physica D 152 (2001), in press.
[7] V. Gurarie, Nucl. Phys. B 569 (1995).
[8] A. Dyachenko, A.C. Newell, A. Pushkarev, V.E. Zakharov, Physica D 57 (1992) 96.
[9] A.C. Newell, V.E. Zakharov, Phys. Rev. Lett. 69 (1992) 1149.
[10] D. Cai, A.J. Majda, D.W. McLaughlin, E. Tabak, Physica D 152 (2001), in press.