MA455 Manifolds
Programme, Aims, Objectives, all rolled into one

Week 1: Submanifolds of Euclidean space. We begin with the example of the torus in $\mathbb{R}^3$, which we parametrise (by a bi-periodic map from the plane) and define by an equation, and the sphere in $\mathbb{R}^3$, where we look at stereographic projection and the crossover maps relating north projection and south projection. In this context we introduce, and later formalise, many of the basic concepts: chart, parametrisation, crossover map, atlas, diffeomorphism, and later tangent vector and tangent space. We keep up our sleeve the interpretation of the torus as a quotient of the plane by a group action/equivalence relation. We stress from the outset that Euclidean space is only a temporary home for Manifolds, and that we will later describe them in a more abstract way (see entry for Week 7). We also revise the inverse and implicit function theorems.

Week 2: Normal forms for submersion and immersions. The preimage of a regular value of a smooth map is a manifold. A proper 1-1 immmersion is a diffeomorphism onto its image. Transversality: the transverse preimage of a submanifold is a submanifold. Submanifolds in general position. Sard's theorem (with sketch of proof).

Week 3: The tubular neighbourhood theorem. Partitions of unity. Applications of Sard's theorem: moving things into general position by isotopies/homotopies. The weak transversality theorem.

Weeks 4-5: Manifolds with boundary. Orientation; boundary orientation, transverse preimage orientation, etc.

Weeks 5-6: Oriented intersection theory. The degree of a smooth map. Homotopy invariance. Oriented intersection numbers. Linking numbers. The Euler characteristic of a compact manifold, as a self-intersection number.

Week 7: Some famous theorems: Brouwer's fixed-point theorem (that every continuous map $B^n \to B^n$ has a fixed-point, where B^n is the unit ball in $\mathbb{R}^n$), the Hairy Ball theorem (that every continuous tangent vector field on the 2-sphere has a zero). Vector bundles. The Poincaré Hopf-theorem (with proof only sketched).

Week 8: Multilinear algebra and alternating tensors. Differential forms and the exterior algebra $\Omega^\bullet(X)$.

Week 9: The exterior derivative. Pull-backs of differential forms by smooth maps. Integration on manifolds.

Week 10: Stokes's theorem. The degree theorem

$$\int_X f^*(\omega) = \deg(f) \int_Y \omega,$$

and Gauss's formula for the linking number.

Books in order of difficulty:

1. J. Milnor, *Topology from the differentiable viewpoint*, University of Virginia Press £6.95. This is a classic, very short, and easy to read. It does not cover all of the material of the course, but by introducing the student to some of the ideas, if not the techniques, it would make a good preparation. *I stongly recommend chapters 1 through 5 as bed-time reading during the first couple of weeks of this module.*

2. V. Guillemin and A. Pollack, *Differential Topology*, Prentice Hall. Excellent but grotesequely overpriced. Begins with the same viewpoint as this module, but does not progress to abstract (non-embedded) manifolds.

3. R.Bott and L.Tu, Differential forms in algebraic topology, Springer Graduate Texts in Maths. A classic, but beginning at a higher level and going well beyond what we will cover.

4. I.Madsen and J. Tornehave, *From calculus to cohomology*, Cambridge University Press, 1997, £16.95 (in 1997). Covers the material on differential forms very well; is one of the texts for a follow-up (MA5N0) to this module. Somewhat technical exposition.