Introduction to Geometry Exercises 2

Aims of these exercises: To develop intuition of geometry on the sphere; to practice writing two-column proofs.

Section A

1. Show that if AB and CD are arcs of great circles on the sphere, and have the same length, then there is an isometry of the sphere taking taking AB to CD.

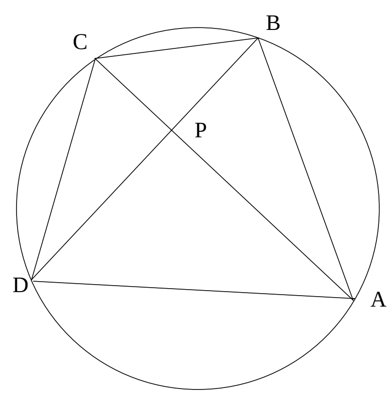
2. (i) Find a spherical triangle with angle sum $3\pi/2$.
(ii) Find a spherical triangle with angle sum greater than 2π.
(iii) How big can you make the angle sum in a spherical triangle?

Many of the exercises from 3 on make use of Exercise 2 on Exercise Sheet 1.

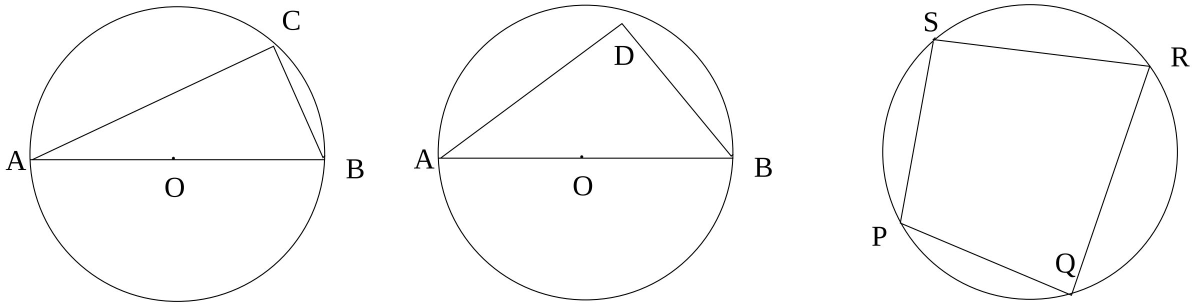
3. Show that the angle $A\hat{C}B$ in the diagram in Exercise 5 below (where O is the centre of the circle) is a right-angle.

Section B

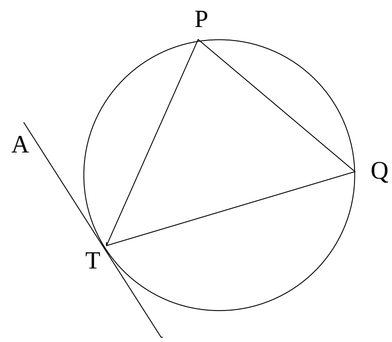
4. Two triangles ABC and PQR are *similar* if $A\hat{B}C = P\hat{Q}R$, $B\hat{C}A = Q\hat{R}P$ and $C\hat{A}B = R\hat{P}Q$. Find two pairs of similar triangles in the diagram below, and prove that they are similar.



5. (i) In the middle diagram below, O is the centre of the circle. Show that if $A\hat{D}B$ is a right-angle, then D in fact lies on the circle. Hint: continue AD until it meets the circle at, say, D'. What can you say about $A\hat{D}'B$?
(ii) Show that opposite angles in a cyclic quadrilateral (e.g. $P\hat{Q}R$ and $R\hat{S}P$ in the picture below) add up to π.



6. Show that $A\hat{T}P = T\hat{Q}P$.



7. Explain how to find a circle passing through three given points on a sphere. Justify your procedure as far as you are able.

Section C

8. If opposite angles in a quadrilateral add up to π, do its four vertices all lie on the same circle? Give a proof or a counter-example.

9. (i) In Exercises 1 we showed that every isometry of the plane is the composite of no more than 3 reflections. How many reflections are needed to obtain a rotation? And a translation?

(ii) Find an isometry of the plane which is not a reflection, a translation or a rotation.

(iii) Suppose that f is an isometry of the plane. How could you tell, by looking at a triangle and its image under f, whether f is the composite of an odd or an even number of reflections?. Experiment until you begin to see what is going on. There is a very simple criterion.