1 Vanishing homology of parameterisations of hypersurfaces

Most of the lecture is based on "Unsolved problems in the theory of singularities of mappings", available online at
http://homepages.warwick.ac.uk/ masbm/LectureNotes.html
Other references are to a book–in–progress available at the same url. This lecture outline is also available there.

1.1 μ versus τ

1. Germs of mappings from n-space to $n+1$-space show "μ, τ"-relation as in ICISS.
2. This relation can be seen already in the three Reidemeister moves of knot theory. The three moves are those unavoidably present when we deform one plane knot diagram to another.

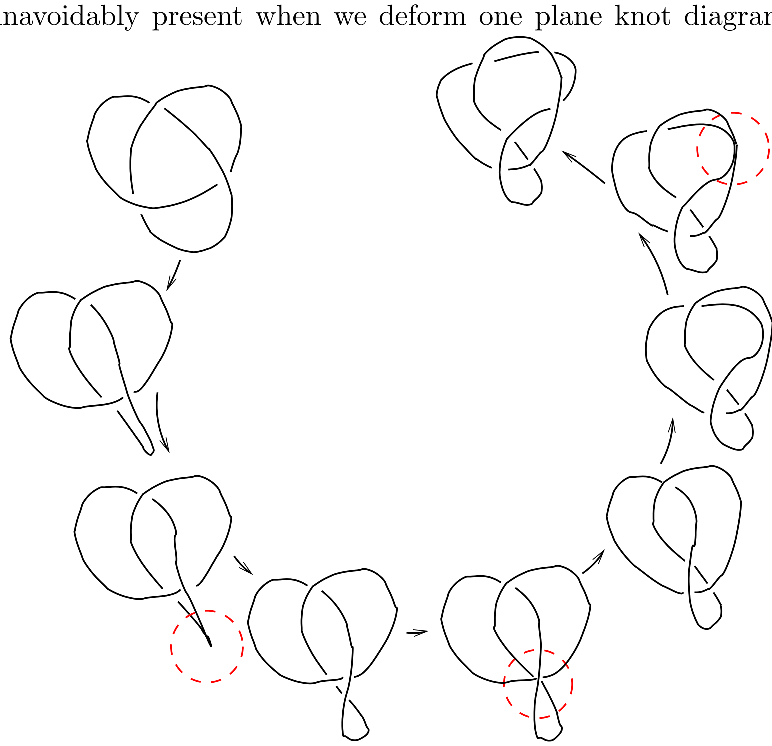


Figure 1: Deforming a planar projection of a trefoil, passing through moves I, III and II

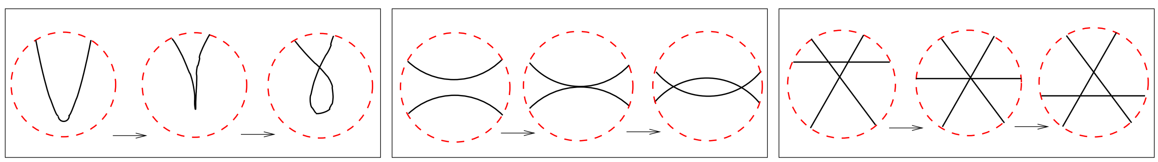
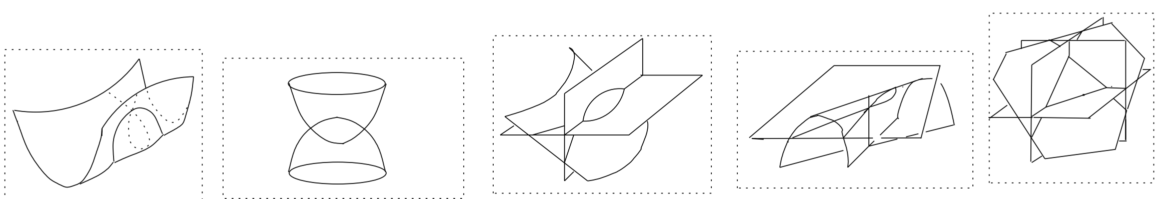


Figure 2: Reidemeister moves I, II and III, isolated in their Milnor balls

3. Definition of Milnor ball. *Book, Theorem 1.1.7 and following page.*

4. In Reidemeister moves, rank of homology of a good real stable perturbation = $\mathcal{A}_e$-codimension.
5. Definition of $\mathcal{A}$-equivalence.
   For 1–5, Unsolved Problems pages 1-2
6. Definition of unfolding, stability and versality;
   Book Chapter 3, pp 29-32
7. Versality criterion: infinitesimal versality implies versality (*Unsolved, Theorem 1.2*)
8. Definition of $\mathcal{A}_e$-tangent space and $\mathcal{A}_e$-codimension. (*Unsolved, pages 2-4*
9. Calculation of $\mathcal{A}_e$-codimension for (i) RI, (ii) node and (iii) RII (iv) RIII as exercise. (*Unsolved, Example 1.1*)
10. Isolated instability $\Leftrightarrow$ $\mathcal{A}_e$-codimension $< \infty \Leftrightarrow f$ is finitely determined. *Book, Section 3.7*
11. Definition of nice dimensions and notion of stable perturbation.
12. A finitely determined germ in the nice dimensions has a stable perturbation, unique up to homeomorphism. (*Book, Section 4.4*)
13. List of stable germs is rather short: for maps $\mathbb{K} \to \mathbb{K}^2$, only immersions and nodes. For maps $\mathbb{K}^2 \to \mathbb{K}^3$, only immersions, double points, triple points and Whitney umbrellas. Mather's theorem on classification by $\mathcal{R}$-algebras , leading to his construction of stable germs as $\mathscr{K}_e$ versal unfoldings (*Book, Chapter 4*). Use of Thom Transversality Theorem (*Book, Section 1.5*) to find candidate stable germs.
14. Calculation of $\mathcal{A}_e$-tangent space for Whitney umbrella $f(x, y) = (x, y^2, xy)$. (*Book, Section 3.1*



15. *Figure 3: Images of stable perturbations of codimension 1 maps from 2-space to 3-space*

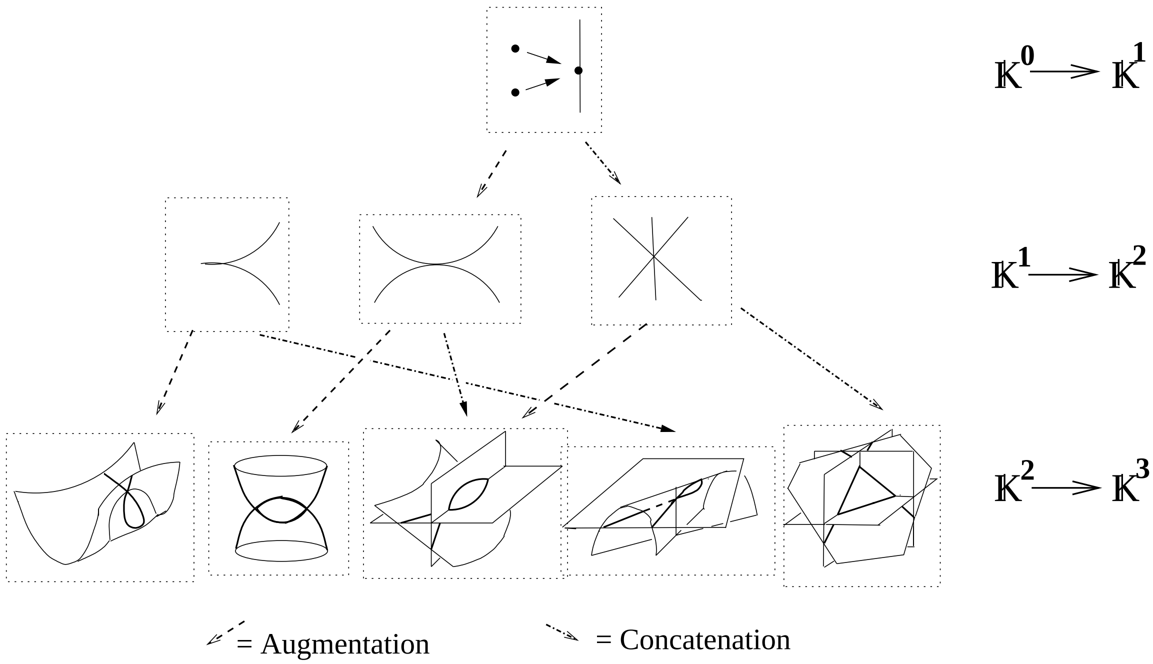


Figure 4: Augmentation and Concatenation generate new codimension 1 germs from old.

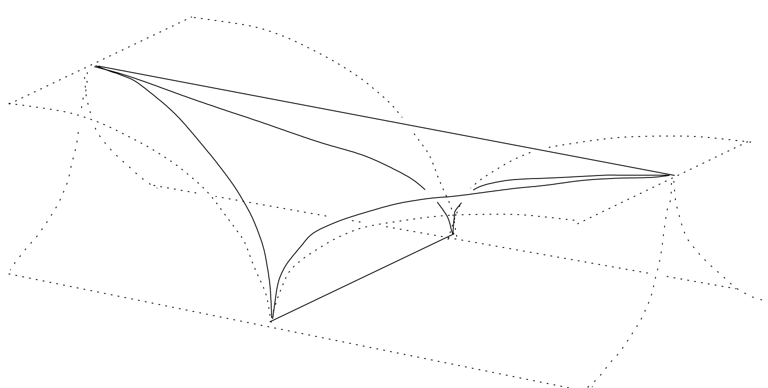


Figure 5: Discriminant of stable perturbation of binary concatenation of two cusp mappings

16. Rank of homology alone not a very good description. Better description in terms of multiple point spaces.