Game Theory and Climate Change

David Mond
Mathematics Institute
University of Warwick

Mathematical Challenges of Climate Change

- Climate modelling involves mathematical challenges of unprecedented complexity. Today's climate models
  - gather data at $\simeq 10^9$ different locations,
  - input it into a complicated partial differential equation,
  - process the results
  - compare with subsequent data,
  - and try to improve the equations
  - to make more accurate long-term predictions.

  They use the world's most powerful computers, and have become steadily more convincing . . . and alarming . . .
- but in the 2012 US electoral campaign, no-one wanted (dared?) to talk about climate change.


I HAVE A NAGGING FEELING THERE'S SOMETHING WE FORGOT TO TALK ABOUT DURING THIS ENTIRE CAMPAIGN.
CLIMATE CHANGE
NEVER MIND THE CAMPAIGN! WE'RE IN THE MIDST OF A HISTORIC, NEVER-BEFORE-SEEN, RECORD-BREAKING SUPERSTORM!
...BESIDES. I HAVE NO CLUE WHAT YOU'RE THINKING OF.
HORSEY
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Mathematical challenges of climate change

- Climate change is a hot potato that politicians don't want to grasp.
  Sensible action is likely to involve unpopular decisions...
  which are hard to take, given the uncertainty in the data
- So understanding uncertainty presents a challenge for mathematics, and democracy

Are these consequences of climate change?

Refugees from Somalia gathered on the border with Kenya. Are they refugees from drought, or from war? Does drought cause war? Is climate change to blame for this drought?

Drowned cabs in New Jersey after Super-storm Sandy

Unprecedented flooding in the Somerset Levels, spring 2014

Is climate change to blame?

Is climate change to blame for these events?

In a complex system, events may have many causes.

If I say
"I'm certain these floods were caused by climate change"
I mean
"Without climate change they would not have happened"
- something impossible to verify.

Probabilistic version:

before climate change,
$P(\text{two month-long floods in the Somerset Levels}) = p_1$.

after climate change,
$P(\text{two month-long floods in the Somerset Levels}) = p_2$.

and $p_2 > p_1$. We need to understand probability theory, but . . .

In a survey carried out by the Royal Statistical Society in 2012

> *a total of 97 UK MPs were asked this probability problem: if you spin a coin twice, what is the probability of getting two heads?*
>
> *Only 38 of the 97 replied correctly, although 72 said they felt confident when dealing with numbers.*

Challenge for scientists: increase our own understanding of probability and uncertainty, and share it.
We need our legislators to understand probabilities!

Mathematical challenges of climate change

- Climate modelling involves mathematical challenges of unprecedented complexity.
- Understanding uncertainty: a challenge for mathematics and democracy
- But hardest of all:

agreeing to do something about it!

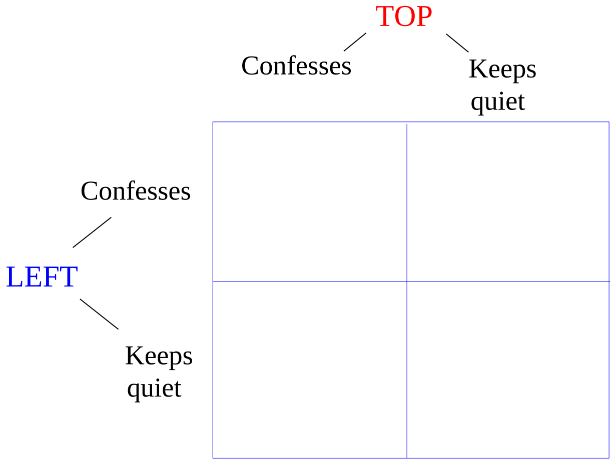
Not just rocket science . . .

Game Theory gives insights into why negotiations fail.

Sample "game": the prisoner's dilemma

Two climate activists, Top and Left, carry out an illegal action and are arrested. They are interrogated separately. Each has 2 options:

so 4 outcomes are possible



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TOP

LEFT	Confesses	Keeps quiet
Confesses	9 years 9 years	10 years 0 years
Keeps quiet	0 years 10 years	2 years 2 years

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What do they do?

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What do they do? What are their incentives?

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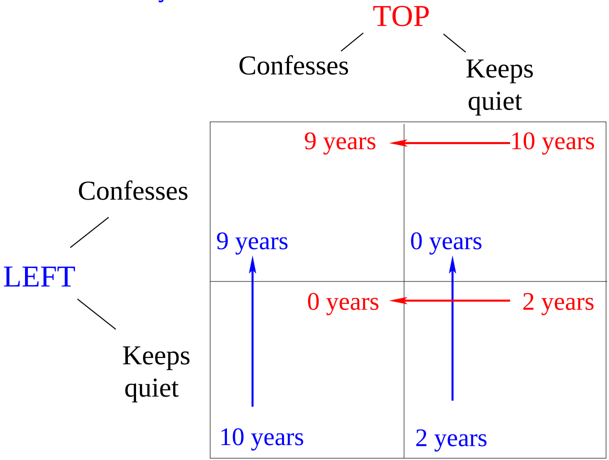


Nash Equilibrium

Whatever Top does, Left does better to confess, and *vice versa*.

In every game, each player has to choose a strategy;

these choices determine the outcome for each.

A set of strategies (one for each player) is a Nash Equilibrium (after the mathematician John Nash (1928-2015))

if once they are adopted, no player can improve his outcome

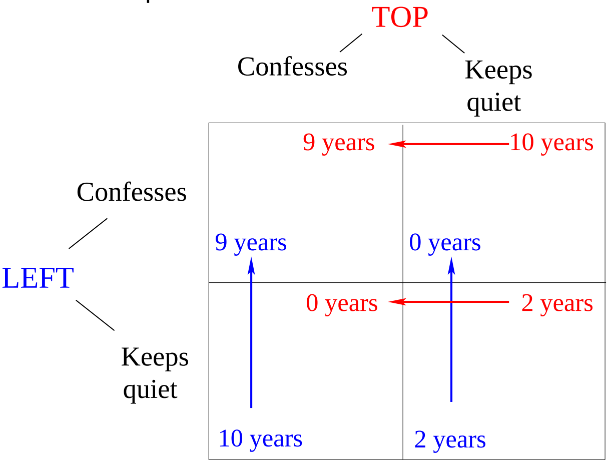
by changing only his own strategy.

So a Nash equilibrium is best for everyone? Not necessarily!

In the prisoner's dilemma,

Top: confesses   Left: confesses

is a Nash equilibrium.



They would have done better to keep quiet!

What's the point of a game?

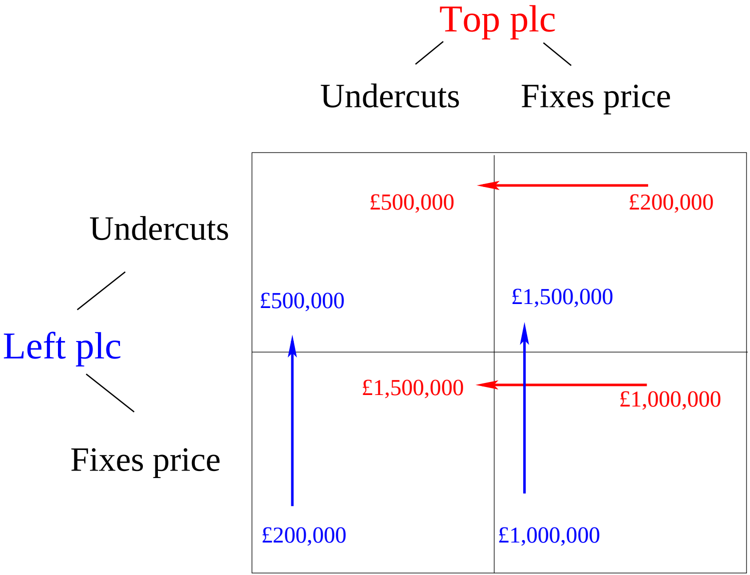
Like all mathematics, Game Theory takes complicated situations and abstracts: simplifies, throws away detail, . . . to reveal underlying structures. We can then see the same structures appearing in many different contexts.

(Change of language: instead of "keeps quiet" or "confesses", we say "cooperates" (i.e. with his fellow activist) or "defects" – terms more generally applicable.)

Example: Two companies compete, selling the same product. If they cooperate, they can both sell at a high price and make big profits. But if one defects by undercutting the other, he will sell more, and do better than his competitor.

Cooperating is called "forming a cartel" in this context, and is against the law. Companies may not communicate their pricing intentions. This means the incentives operate just as in the prisoner's dilemma, in this case to the advantage of the public.

Healthy competition ≃ Prisoner's Dilemma



Nash's work in Game Theory

Theorem: Every game has a Nash equilibrium, if mixed strategies are allowed.

(Optional explanation :A "mixed strategy" plays each strategy S_i with probability p_i.

The payoff is now the "expected value"

$$\sum_i p_i \; Payoff\,(S_i). \qquad\qquad)$$

This theorem and other related work (mathematically the simplest thing he ever did) earned Nash the Nobel prize for Economics in 1994.

Over 40 years it had revolutionised the field.

How do players arrive at a Nash equilibrium?

Not necessarily through rational evaluation of the available strategies.

Players in repeated games may reach a Nash equilibrium through trial and error – or just plain error –

and it may be hard to escape from.

Evolutionary Game Theory studies how this occurs – and how 'players' (societies, species, political parties, competing companies . . .) can sometimes avoid falling into a damaging Nash equilibrium, for example by evolving altruism.

Sub-optimal Nash equilibria in Public Goods games

Key idea: players reap benefits of their actions for themselves, but share the costs among many ...

... Privatise the gain, share the pain. ...

1. "Tragedy of the Commons" Villagers graze their animals on shared common land. If I graze my animals,

Benefit (all to me): My animals grow fat

Cost (shared among all): Grass is eaten

Outcome if we all do it: Overgrazing and loss of shared resource

Sub-optimal Nash equilibria in Public Goods games

2. Downward wages spiral

Companies compete by downsizing and lowering the wages of their employees

Benefit: I reduce my production costs $\Rightarrow$ I charge

lower prices $\Rightarrow$ I gain higher market share and more profits

Costs : My workers' purchasing power is reduced; the resulting loss of sales is shared among all companies, so I suffer only a small part of this loss of business.

So it is in each company's individual interest to downsize.

Outcome: All companies do the same $\Rightarrow$ economic downturn

Sub-optimal Nash equilibria in Public Goods games

3. The diner's dilemma, or, How we ate the world

Six friends dine together in a restaurant, agreeing to share the bill equally, but making their choices individually. Two meal options:

Quite expensive meal m

expense	e
pleasure	p

Very expensive meal M

expense	E
pleasure	P

Which to choose?

Suppose $E > P > p > e$

if alone, I would choose m.

In company, the extra cost, to me, if I choose M is

$$\frac{E-e}{6}$$

So if

$$P > e + (E-e)/6$$

I choose M.

Increased benefit (all to me): $P - p$

Increased cost to me $\frac{1}{6}(E-e)$

We all choose M.

We all choose a meal that we would have preferred not to have to pay for... As soon as we leave the restaurant we regret it!

Stupid! But if I don't have the expensive meal, everyone else will, and I will still have to pay. So I opt for M.

Now imagine there are 7.3 billion diners.

There are!

Fortunately, not everyone attends climate negotiations; still, more than 100 countries participate.

Climate negotiators are like the diners:

if I don't continue burning fossil fuels, everyone else will, and I will still suffer climate change, and worse still, with a weaker economy due to my reduced use of fossil fuels. So no treaty is agreed.

The strategy of postponing action is a malign Nash equilibrium.

How to leave a sub-optimal Nash Equilibrium ?

Alliances change the nature of the game.

1. In Prisoner's dilemma, if the two climate activists have faith in one another, they can both keep quiet.
2. If villagers agree to limit grazing, a tragedy of the commons can be averted. This happens naturally in small communities where everyone knows everyone else: public disapproval of over-use of the shared resource can be sufficient disincentive.
3. A trades union, or a minimum wage, which ensures wages are not depressed, can avoid a downward wages spiral.
4. Almost all cooperative action requires sanctions against freeloaders.

Discuss: What strategies of common action enable diners who share the bill to moderate their consumption?

Understanding that a Nash equilibrium is not necessarily optimal can help negotiators seek alliances.

Members of parliament responsible for later ratification of treaties also need this understanding.

And voters have to support them.

It's a tall order! There's a lot of work to do . . .

“The tragedy of the commons”

(Garret Hardin, Nature, 1968).

“We can make little progress in working toward optimum population size until we explicitly exorcise the spirit of Adam Smith in the field of practical demography. In economic affairs, *The Wealth of Nations* (1776) popularized the invisible hand, the idea that an individual who intends only his own gain, is, as it were, led by an invisible hand to promote the public interest. Adam Smith did not assert that this was invariably true, and perhaps neither did any of his followers. But he contributed to a dominant tendency of thought that has ever since interfered with positive action based on rational analysis, namely, the tendency to assume that decisions reached individually will, in fact, be the best decisions for an entire society. If this assumption is correct it justifies the continuance of our present policy of laissez faire in reproduction. If it is correct we can assume that men will control their individual fecundity so as to produce the optimum population. If the assumption is not correct, we need to reexamine our individual freedoms to see which ones are defensible.”

References

Ken Binmore, *A very short introduction to Game Theory*, Oxford University Press 2007

Andrew Dessler, *An introduction to modern climate change*, Cambridge University Press, 2012

Uri Gneezy, Ernan Haruvy, Hadas Yafe, *The inefficiency of splitting the bill*, The Economic Journal, 114 (April 2004) , 265-280

James Hansen, *Storms of my grandchildren*, Bloomsbury, 2009

Garrett Hardin, *The Tragedy of the Commons*, Science, 162 (1968): 1243-1248.