

The groups of order p^7

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Eamonn O'Brien and I have classified the groups of order p^7 , and have built a Magma database of the groups. The 2328 groups of order 2^7 were found by James, Newman and O'Brien in 1990. There are 9310 groups of order 3^7 and 34297 groups of order 5^7 . For $p > 5$ the number of groups of order p^7 is

$$\begin{aligned} &3p^5 + 12p^4 + 44p^3 + 170p^2 + 707p + 2455 \\ &+ (4p^2 + 44p + 291) \gcd(p-1, 3) + (p^2 + 19p + 135) \gcd(p-1, 4) \\ &+ (3p + 31) \gcd(p-1, 5) + 4 \gcd(p-1, 7) + 5 \gcd(p-1, 8) + \gcd(p-1, 9). \end{aligned}$$

For $p > 5$, we actually classify the nilpotent Lie rings of order p^7 , and then use the Baker-Campbell-Hausdorff formula to translate Lie ring presentations into group presentations.

We make extensive use of Magma programs both in classifying the Lie rings of order p^7 , and in checking the accuracy of our results.