Learning to think flexibly in mathematics using Japanese Lesson Study

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A Long-Term Learning Framework for Lesson Study

For many individuals, mathematics is *complicated* and it gets more complicated as new ideas are encountered.

For others, by focusing on the *essential ideas*, it becomes possible to see mathematics in a more focused way that makes many ideas essentially more *simple*.

"Technical skill is mastery of complexity, while creativity is mastery of simplicity."

(Sir Christopher Zeeman)

The question is, how can we help each and every child find this simplicity in a way that works *for them*?

Symbolic Compression

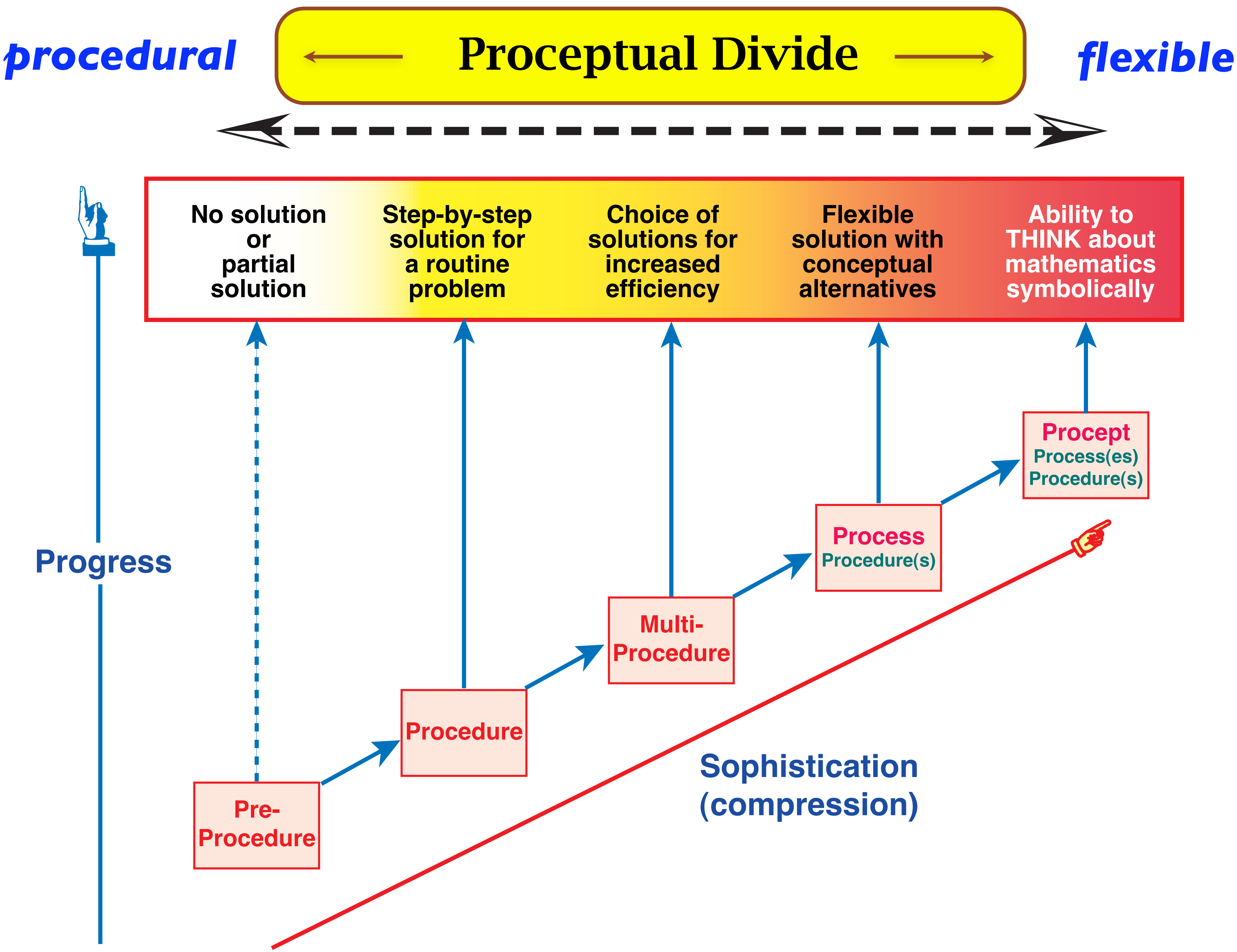
(i) *Procedure*: A step-by-step procedure to carry out the operation;

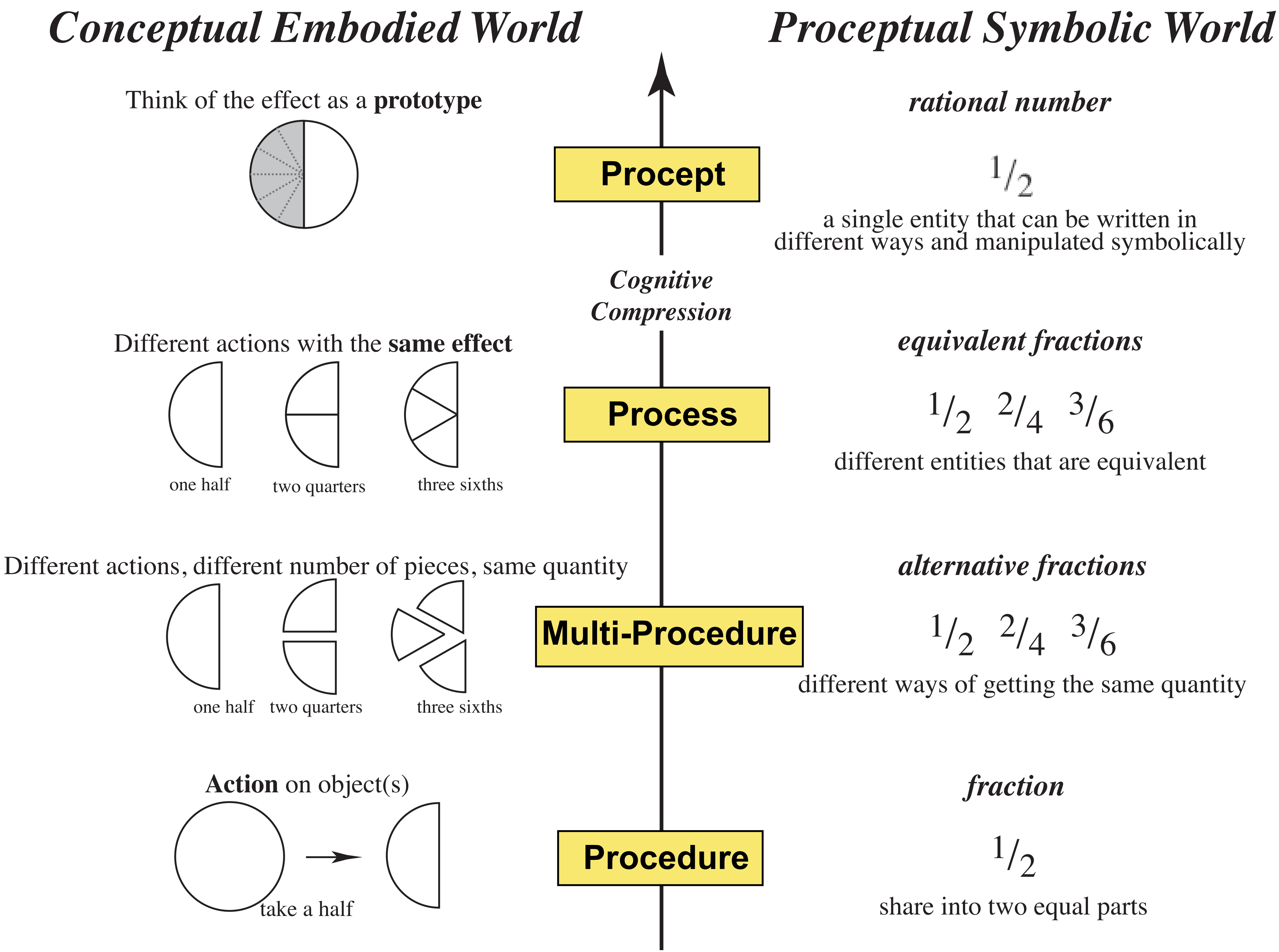
(ii) *Multi-Procedure*: Several different procedures to carry out the same operation, allowing a choice of the most efficient;

(iii) *Equivalent Procedures*: Different procedures may involve different sequences of steps, but they all achieve 'the same result' and are seen as 'equivalent';

(iv) Procept: The realisation of a single underlying concept that can be represented flexibly by different symbols.

Symbolic Compression





Conceptual Embodied World
Proceptual Symbolic World
Think of the effect as a **prototype**
rational number
Procept
1/2
a single entity that can be written in different ways and manipulated symbolically
Cognitive Compression
Different actions with the **same effect**
equivalent fractions
Process
one half
two quarters
three sixths
1/2 2/4 3/6
different entities that are equivalent
Different actions, different number of pieces, same quantity
alternative fractions
Multi-Procedure
one half
two quarters
three sixths
1/2 2/4 3/6
different ways of getting the same quantity
Action on object(s)
fraction
Procedure
take a half
1/2
share into two equal parts

Symbolic Compression

Multi-digit multiplication

How do we teach the procedure of long-multiplication?

As a single procedure of column multiplication?

As the best of several procedures?

How do we relate the ideas together and help each child give individual meaning by building on their experiences?

Symbolic Compression

Multi-digit multiplication

The Japanese idea of Ha-Ka-Se.

Ha-Ka-Se means ‘professor’

Hayai means ‘fast’

Kantan means ‘easy’ (and understandable).

Seikaku means accurate (and logical)

Method: Get the children to suggest different methods to select the **fastest**, **easiest**, and **most accurate.**

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Example: Introducing Multi-digit Multiplication

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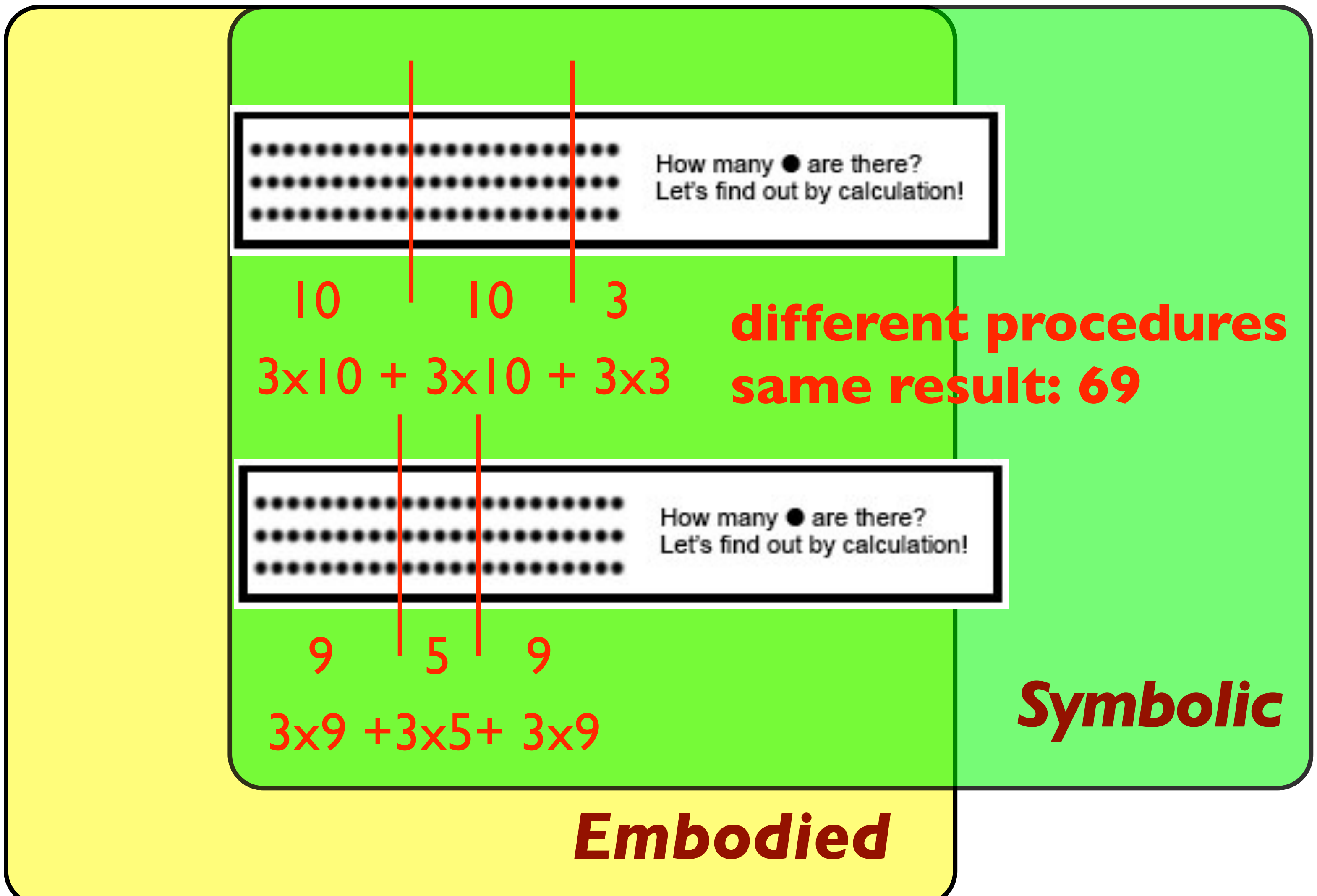

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Example: Introducing Multi-digit Multiplication

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Method:

1. Embody the product 23x3
2. Find *several different ways* of calculation
so that 23x3 is the same as (10+10+3)x3
or as (9+9+5)x3 or (20+3)x3
(*embodiment* giving meaning to symbols)
3. See *flexibility*, that *all of these are the same*
4. See the standard algorithm is the most efficient
5. Relate this to the standard algorithm
which is now seen as quick, easy, accurate
within a meaningful context
relating embodiment and flexible symbolism

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Multiplication Algorithm (Grade 3)

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Building ideas in a flexible manner.

Met-before: single-digit multiplication subdividing a problem into smaller problems

Activity: constructing different ways of calculating 3 times 23

Long-term: flexible thinking about multiplication, revealing the standard algorithm as the most efficient.

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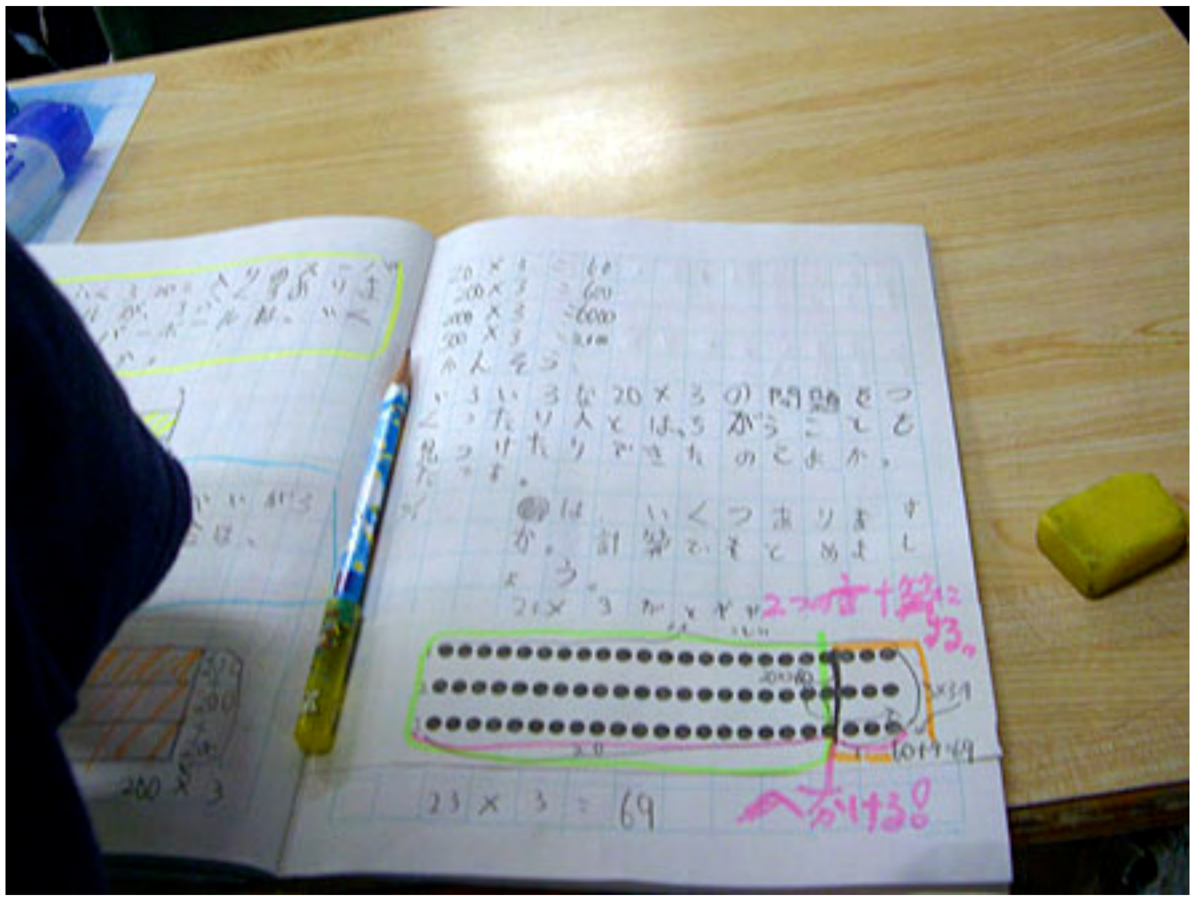

Experimenting with the problem

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Discussing ideas

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Explaining to the teacher

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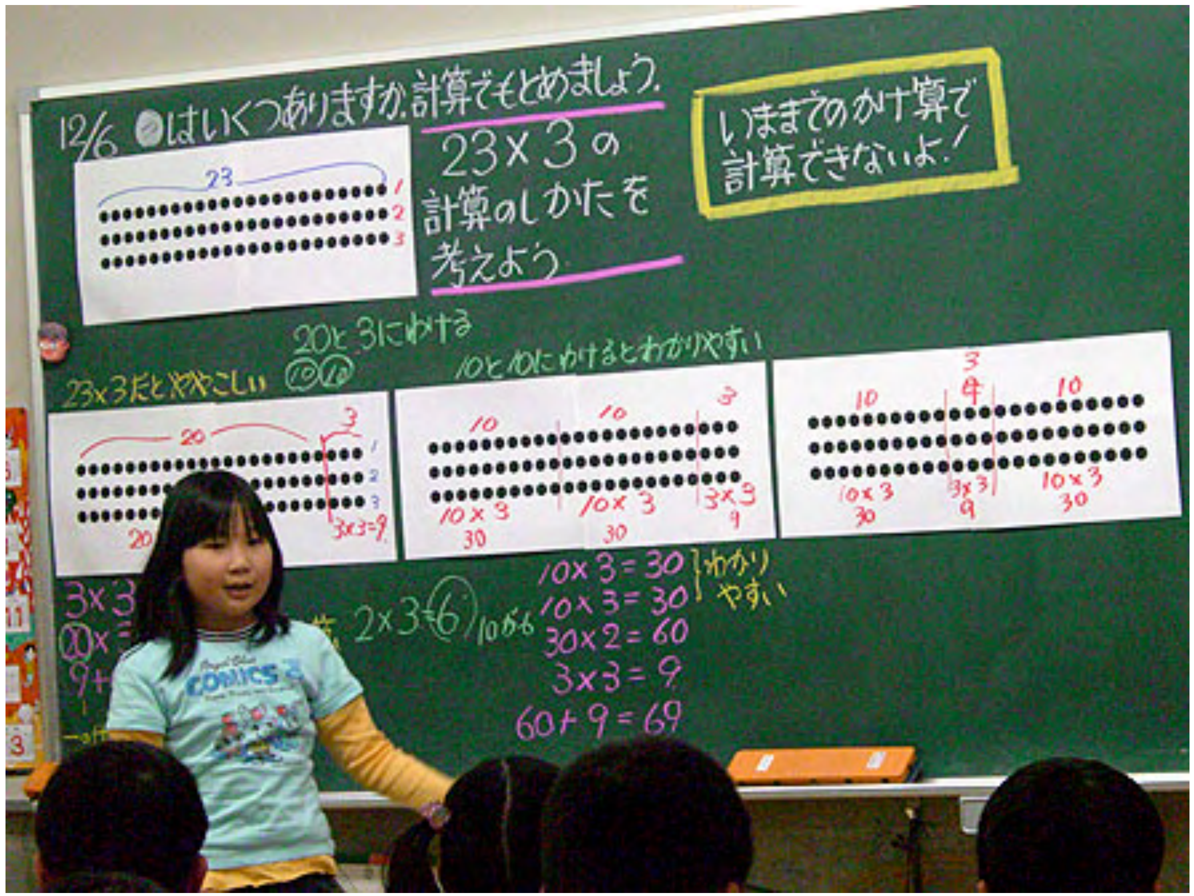

Displaying different solutions

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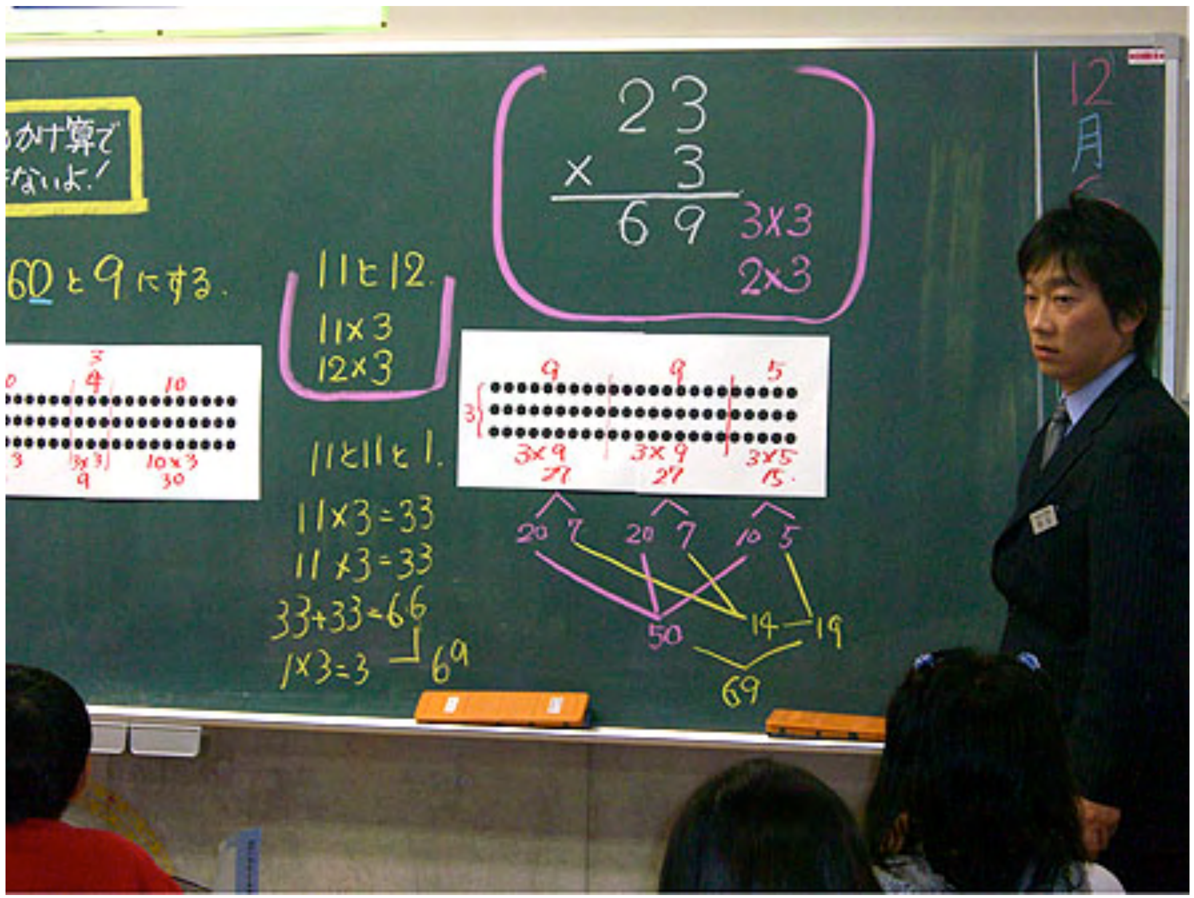

Comparing solutions for efficiency

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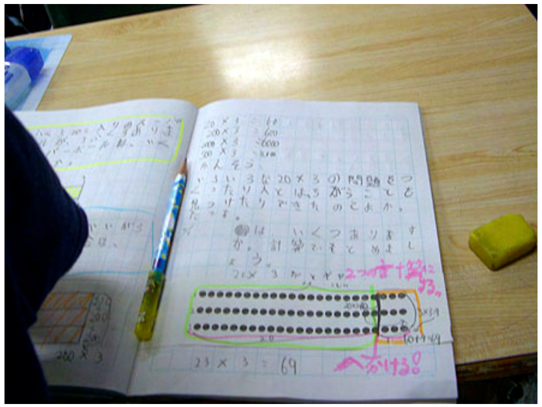

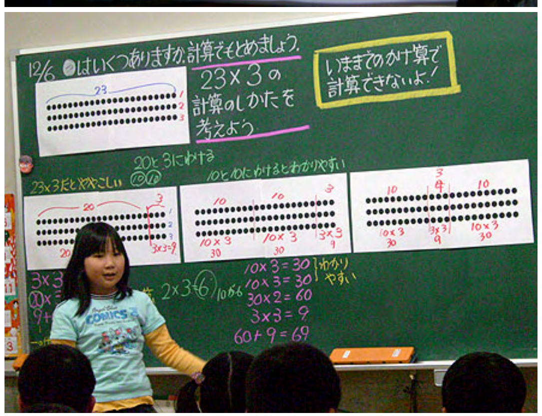

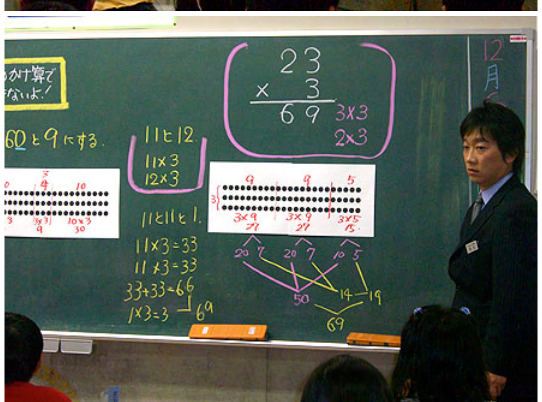

A well-organised lesson in a sequence designed to give a flexible insight into multiplication.

What is the contribution to future development? Different children brought different met-befores. Some struggled with the arithmetic, some already knew the long-multiplication algorithm.

Questions:
What is the *important long-term role* of this lesson that the children should focus on?
What do individual children learn from this experience in the long-term?

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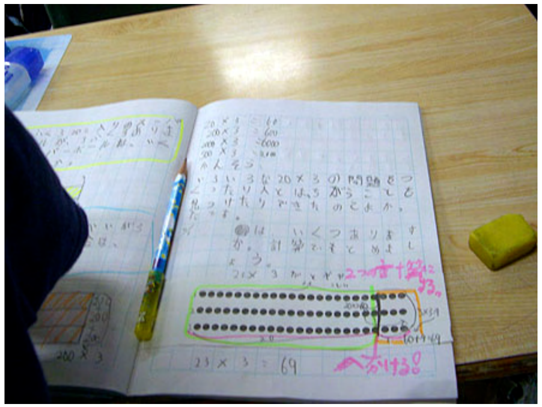

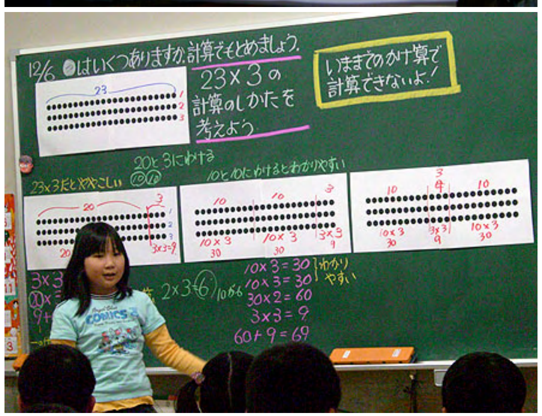

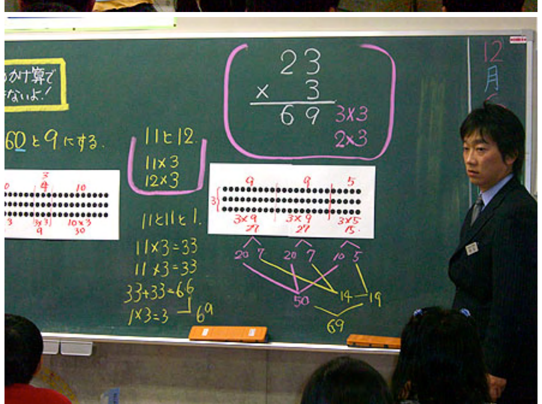

Goals of the Unit:

- Lessons that enable students to consciously think about the connection between what they learned before and what they are learning now
- Lessons in which students learn from each other and that help them consciously think about their own solution processes
- An evaluation method that helps foster students' logical thinking abilities.
- Unit plan
- This lesson (goals, process of lesson)

Questions:

What is the *important long-term role* of this lesson that the children should focus on?

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