Students' Concept Images for Period Doublings as Embodied Objects in Chaos Theory

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The aim of this research is to study how visualisation using dynamic computer software helps students construct conceptual knowledge in a context where symbolic proofs are difficult or unknown and numerical computation offers insightful data. We focus on the study of period doubling in chaos theory, situating our analysis in process-concept theory (Tall et al, 2000), relating this to the embodied theory of Lakoff and Nunez (2000).

The results presented here will reveal in what ways able mathematics undergraduates interpret visual pictures of the eventual behaviour of the iteration of $x=\lambda x(1-x)$ for various values of λ starting at $x \in]0,1]$. For $0< \lambda\leq 3$ the iterations home in on a limit which bifurcates to an iteration of period 2 after $\lambda_0=3$, then to periods 4, 8, … at a sequence of increasing values λ_1, λ_2, … . which seem to approach a limit λ_∞ (the Feigenbaum limit). After this point the behaviour is chaotic and graphing the set of limit points against the corresponding value of λ gives a picture termed the Feigenbaum tree.

The investigation is performed with a second experiment that involves period doubling of a closed loops on an oscilloscope. (See Chae & Tall, 2001 for details.)

In this paper we focus on students' responses to the following post-test questions:

What first comes to your mind when you think about 'period doubling'?
Please draw/make an example of a period doubling in your mind's eye.

The pictures were diverse, including sketches of both of $x=f(x)$ iteration and loop doubling, which we analysed in terms of a new perspective of 'base object'–'process'–'concept'. We found most of the 20 students focused on the pictures as *base objects*; 7 drew a single picture representing the base object *after* bifurcation, 12 drew 'before and after' representing the *process* of bifurcation and only one drew a Feigenbaum diagram which we interpret as moving towards an overall view of the *concept* of bifurcation. Further analysis reveals that of the 8 students drawing loops, 7 concentrate on the move from a single loop to a double loop, but of the 10 students using $x=f(x)$ iteration only one draws the shift from a point to a 2-orbit and only one draws a 2-orbit. We will discuss how this intimates that students focus more on *visual* concepts as embodied objects than on symbolic/numeric representations at this stage of their investigation.

The full paper may be found on the world wide web at http://warwick.ac.uk/staff/David.Tall.

References

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