Professors' perceptions of students' mathematical thinking: Do they get what they prefer or what they expect?

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*In a previous study (Mohd Yusof & Tall, 1994), it was shown that university students in a problem-solving course developed positive attitudes towards mathematics as a process of thinking rather than as a procedural body of knowledge. In this study their teachers are asked to specify the attitudes they **expect** from their students and the attitudes they **prefer**. The difference is used to define the professors' "desired direction of change". It is found that almost all attitudinal changes in the problem-solving course are in the **desired** direction. Six months after returning to standard mathematics lecturing, almost all changes are in the **opposite** direction – consistent with the hypothesis that professors get what they expect, not what they prefer.*

Mohd Yusof & Tall (1994) studied the attitudinal changes in 44 students following a course in mathematical problem-solving based on the approach of Mason *et al* (1982). (There were 24 male and 20 female students – a mixture of third, fourth and fifth year undergraduates aged 18 to 21 studying Industrial Science (majoring in Mathematics) and Computer Education.at Universiti Teknologi Malaysia.) The original study used a 17 item attitudinal questionnaire and showed that students' attitudes to mathematics and problem solving changed in what was considered a positive manner. In particular students' attitudes changed from mathematics as a body of procedures to be memorised to mathematics as a process of thinking.

Here we collect data from the students' teachers to establish their "desired direction of attitudinal change" and further data from the students in a delayed post-test, after six months of standard mathematics lectures. This allows a comparison to be made between the staff's desired change and the actual changes occurring in the students during problem-solving and during a return to regular mathematics teaching. The data from the questionnaires is supplemented by interviews with selected students and staff.

The "desired direction of attitudinal change" perceived by mathematics staff

Members of the Mathematics Department were invited to fill in the attitudinal questionnaire of Mohd Yusof & Tall (1994) twice. On first reading, they were requested to tick the response they ***expect*** from a typical student. On the second they were requested to put a circle where they **prefer** it to be. Twenty-two members of the department took part, responding to the following questionnaire on a five point scale:

Y, y, –, n, N (definitely yes, yes, no opinion, no, definitely no).

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Section A : Attitudes to Mathematics	Section B : Attitudes to Problem-Solving
1. Mathematics is a collection of facts and procedures to be remembered.	1. I feel confident in my ability to solve mathematics problems.
2. Mathematics is about solving problems.	2. Solving mathematics problems is a great pleasure for me.
3. Mathematics is about inventing new ideas.	3. I only solve mathematics problems to get through the course.
4. Mathematics at university is very abstract.	4. I feel anxious when I am asked to solve mathematics problems.
5. I usually understand a new idea in mathematics quickly.	5. I often fear unexpected mathematics problems.
6. The mathematical topics we study at university make sense to me.	6. I feel the most important thing in mathematics is to get correct answers.
7. I have to work very hard to understand mathematics.	7. I am willing to try a different approach when my attempt fails.
8. I learn my mathematics through memory.	8. I give up fairly easily when the problem is difficult.
9. I am able to relate mathematical ideas learned.	

Table 1 : Attitudinal questions to mathematics and problem-solving

Table 2 shows the responses of 22 lecturers in the Mathematics Department and the direction of the desired change from the expected to the preferred attititude. The columns marked “Yes(Y)” have the “total yes” responses (Y+y), with the subset “definitely yes” (Y) in brackets. Similarly for “No(N)”.

				Expect			Prefer		
	Attitude	desired change		Yes (Y)	–	No (N)	Yes (Y)	–	No (N)
Mathematics	facts and procedures	$\downarrow_{+}^{+++}$	<1%	20 (8)	0	2 (0)	13 (4)	0	9 (2)
	solving problems	$\uparrow_{+++}^{+++}$	n.s.	19 (9)	0	3 (0)	22 (9)	0	0 (0)
	inventing new ideas	$\uparrow_{-}^{+}$	n.s.	8 (2)	0	14 (1)	11 (3)	0	11 (1)
	abstract	$\downarrow_{--}^{+++}$	<1%	20 (6)	0	2 (0)	7 (0)	0	15 (4)
	understand quickly	$\uparrow_{---}^{+}$	<1%	3 (0)	0	19 (6)	15 (1)	0	7 (1)
	make sense	$\uparrow_{-}^{++}$	<1%	8 (0)	0	14 (2)	19 (3)	0	3 (0)
	work very hard	$\downarrow_{++}^{+++}$	n.s.	21 (13)	0	1 (0)	18 (4)	0	4 (0)
	memorisation	$\downarrow_{---}^{++}$	<1%	15 (5)	0	7 (1)	2 (1)	0	20 (6)
	ability to relate ideas	$\uparrow_{--}^{+++}$	<1%	5 (0)	0	17 (5)	22 (5)	0	0 (0)
Problem Solving	confidence	$\uparrow_{-}^{+++}$	<1%	10 (1)	0	12 (0)	22 (3)	0	0 (0)
	pleasure	$\uparrow_{+}^{+++}$	n.s.	15 (0)	0	7 (2)	21 (4)	0	1 (0)
	only to get through	$\downarrow_{-}^{+++}$	<1%	21 (9)	0	1 (0)	7 (2)	0	15 (3)
	anxiety	$\downarrow_{---}^{++}$	<1%	16 (5)	0	6 (0)	2 (0)	0	20 (5)
	fear unexpected	$\downarrow_{--}^{++}$	<1%	15 (7)	0	7 (0)	3 (0)	0	19 (5)
	correct answers	$\downarrow_{-}^{++}$	<1%	19 (3)	0	3 (0)	6 (2)	0	16 (2)
	try new approach	$\uparrow_{+}^{+++}$	<1%	12 (1)	0	10 (0)	22 (4)	0	0 (0)
	give up	$\downarrow_{--}^{++}$	<5%	16 (2)	0	6 (0)	2 (0)	0	20 (2)

Table 2 : Lecturers’ perceptions of students preferred and expected attitudes

The arrow and the plus and minus signs in the second column indicate the direction of movement. The number of plus or minus signs indicates the average weighted strength of response, taking each Y response as 2, y as 1, n as –1 and N as –2. If the average response is 1 or more, the response is considered "strong" and denoted +++ or ---. Between 0.5 and 1 it is denoted "++" or "--", and less than 0.5 it is considered "weak" denoted "+" or "-". For instance, "facts and procedures" is desired to change down from an expected strong agreement (+++) by the typical student to a preferred weak agreement (+) by the lecturers. In line 4, "being abstract" diminishes from an expected strong agreement (+++) to a preferred disagreement (--). The significance of the change is computed using a χ^2 test (with Yates correction) on the number of yes responses (Y+y) and is given as significant (<5%), highly significant (<1%) or not significant (n.s.).

In only four of the cases is the change too small to be statistically significant: the lecturers expect the typical student to believe strongly that ***mathematics is about solving problems*** and prefer it marginally stronger, that ***mathematics is not about inventing new ideas***, but weakly prefer that it should be, that the student has a strong expectation to ***have to work hard to understand***, whilst lecturers have a lower expectation, and that there is ***a weak expectation of pleasure***, but lecturers prefer it to be strong.

One change in direction is statistically significant – that the typical student is expected to ***give up when a problem gets difficult***, but the lecturers prefer the opposite.

Two differences remain in the same direction but the changes are highly significant – an expected strong student belief that ***mathematics is a collection of facts and procedures to be remembered***, which the lecturers desire less, and ***a weak expectation that they are willing to try a different approach*** when their attempt fails, which is preferred stronger.

The remaining ten are both statistically highly significant and have opposite expectances and preferences. The lecturers *expect* the typical student to think mathematics is very ***abstract***, will ***not understand quickly***, will consider that mathematics does ***not make sense***, will ***learn through memory***, will ***not relate mathematical ideas***, will ***not have confidence***, will only solve problems ***to get through*** the course, will show ***anxiety***, will ***fear*** the unexpected, and regard ***correct answers*** as the most important thing. In every case the lecturers *prefer* the student to think the opposite.

Individual interviews with lecturers

Interviews revealed substantial differences in meaning of ideas expressed in the questionnaire from the ideas of "mathematical thinking" in the problem-solving course. For instance, Kilpatrick & Stannic (1989) suggest three different perceptions of problem solving—as means to a focused end, as skill and as art. It soon became apparent that the lecturers see it more as a means to achieve a specific end or a skill to be learned rather than the art of thinking mathematically. "Inventing new ideas" was perceived as original research rather than just ideas new to the individual, as in the following quotation:

> To me mathematics is a tool for solving problems. One way of motivating the students is by showing them applications in the real world. In this way they get the knowledge and the skills for solving problems. … I do not think the students are capable of creating new ideas on their own.

Lecturers are not certain of the problem-solving techniques used in the course:

> … I am not sure of these [processes]. I have not thought about them and I don't know how to go about [teaching] them. I think I need to learn more about them before I can implement them. We developed certain abilities to look at problems but we are not sure how those abilities came to be with you.

Instead they show students how to do examples in the hope that they will develop their own techniques:

> The experience of making conjectures, generalising and the like, I think the students can get themselves on their own, from doing their project work. We do not have the time to teach them everything.
>
> We tell them how to do it – for example, what are the criteria that should be fulfilled in the formula before they can use it. Normally I explain only part of it then I think the students can complete it themselves. … I think that is sufficient for the students.
>
> Under the circumstances, I expect students to acquire the mathematical skills and to get high marks in the exam. … I would want them to become good problem solvers but I am not sure they would be. I myself did not try to get them into becoming one consciously.

Some lecturers genuinely want to change the system but are not sure how to do so:

> I would like students not only to see mathematics as a subject that they need to learn and pass in an exam but also as a discipline which enables them to think for themselves. My main aim is not in trying to finish the syllabus but rather in making the students learn the mathematics in a more meaningful way. … I am not really sure how but I am trying to do it.
>
> To me mathematics is a mental activity but I should say that at this level I presented it more as a formal system. Because we are confined by the syllabus and also depending on the students' background. … I would like it to change. How do I do that? I don't know.
>
> There are a lot of problems that we face. Firstly the students themselves do not have the motivation in their mathematics learning. Secondly they do not have the confidence in their ability to do mathematics. So we have to deal with these first before we can make them see mathematics as a thinking subject.
>
> I very rarely allow students to think [mathematically]. The problems that we gave them do not require them to use their thinking capability. … It is due to the shortness of time.
>
> We give them little room to do their own thinking. But we cannot change it because the system does not allow us to do so. So we end up teaching them what they need to know.
>
> The system has been proven a failure. It has not been successful in producing good mathematicians, or engineers that can use mathematics effectively. They only know how to use procedures or computer packages without really understanding why they use them. …It's all down to the system. We are not training students to discover patterns, or how to prove a statement is true, for example. What we teach them is mainly how to use the procedures.

The change in student attitudes in problem solving and mathematics lectures

To discover how the attitudes of the students changed, the same attitudinal questionnaire was given before and after the Problem-Solving course, then six months later after a semester of standard mathematics lectures. The responses were as follows:

		Before P S			After P S			After Math		
		Yes (Y)	No (N)	–	Yes (Y)	No (N)	–	Yes (Y)	No (N)	–
Mathematics	facts and procedures	34 (18)	8 (2)	2	11 (3)	32 (8)	1	30 (9)	14 (1)	0
	solving problems	27 (10)	16 (4)	1	42 (21)	0 (0)	2	32 (22)	12 (0)	0
	inventing new ideas	21 (4)	21 (6)	2	37 (15)	5 (0)	2	24 (4)	18 (1)	2
	very abstract	25 (13)	17 (0)	2	15 (8)	26 (3)	2	22 (11)	21 (0)	1
	understand quickly	9 (0)	30 (5)	5	20 (3)	21 (2)	3	13 (2)	29 (1)	2
	make sense	22 (4)	22 (5)	0	35 (5)	7 (0)	2	29 (4)	14 (0)	1
	work very hard	37 (15)	5 (1)	2	28 (8)	13 (0)	3	32 (8)	12 (1)	0
	learn by memory	30 (1)	12 (2)	2	11 (0)	31 (7)	2	20 (2)	22 (1)	2
	able to relate ideas	24 (8)	14 (2)	2	35 (11)	8 (0)	1	31 (5)	10 (0)	3
Problem Solving	confidence	26 (7)	17 (2)	1	36 (12)	6 (0)	2	34 (7)	10 (0)	0
	pleasure	43 (25)	1 (1)	0	42 (21)	0 (0)	2	42 (21)	1 (0)	1
	get through	16 (4)	27 (8)	1	4 (0)	37 (17)	3	14 (1)	29 (5)	0
	anxiety	17 (1)	24 (4)	3	6 (0)	36 (9)	2	9 (0)	32 (4)	2
	fear unexpected	30 (10)	12 (3)	2	10 (3)	31 (9)	3	16 (3)	28 (2)	0
	correct answers	21 (4)	21 (3)	2	5 (1)	36 (11)	3	17 (0)	23 (7)	2
	try new approach	42 (17)	0 (0)	2	43 (20)	0 (0)	1	43 (16)	1 (0)	0
	give up	19 (3)	24 (9)	1	5 (0)	37 (20)	2	8 (0)	34 (12)	2

Table 4 : The changing attitudes of students before and after problem-solving and “after math”

Calculating the significance in the change of the total “yes” responses and using a weighted average response as in table 3, we find the following changes:

		desired change		After P S		After math		Total change	
Mathematics	facts and procedures	$\downarrow_{+}^{+++}$	<1%	$\downarrow_{--}^{++}$	<1%	$\uparrow_{--}^{++}$	<1%	$\downarrow_{++}^{++}$	n.s.
	solving problems	$\uparrow_{+++}^{+++}$	n.s.	$\uparrow_{+}^{+++}$	<1%	$\downarrow_{++}^{+++}$	<1%	$\uparrow_{+}^{++}$	n.s.
	inventing new ideas	$\uparrow_{-}^{+}$	n.s.	$\uparrow_{-}^{+++}$	<1%	$\downarrow_{+}^{+++}$	<1%	$\uparrow_{-}^{+}$	n.s.
	very abstract	$\downarrow_{--}^{+++}$	<1%	$\downarrow_{-}^{+}$	n.s.*	$\uparrow_{-}^{+}$	n.s.	$\downarrow_{+}^{+}$	n.s.
	understand quickly	$\uparrow_{---}^{+}$	<1%	$\uparrow_{--}^{o}$	<1%	$\downarrow_{-}^{o}$	n.s.	$\uparrow_{--}^{-}$	n.s.
	make sense	$\uparrow_{-}^{++}$	<1%	$\uparrow_{-}^{++}$	<1%	$\downarrow_{+}^{++}$	n.s.	$\uparrow_{-}^{+}$	n.s.*
	work very hard	$\downarrow_{++}^{+++}$	n.s.	$\downarrow_{+++}^{+++}$	n.s.*	$\uparrow_{++}^{++}$	n.s.	$\downarrow_{++}^{+++}$	n.s.
	learn by memory	$\downarrow_{---}^{++}$	<1%	$\downarrow_{--}^{+}$	<1%	$\uparrow_{--}^{-}$	<5%	$\downarrow_{-}^{+}$	n.s.*
	able to relate ideas	$\uparrow_{--}^{+++}$	<1%	$\uparrow_{+}^{++}$	<5%	$\downarrow_{++}^{++}$	n.s.	$\uparrow_{+}^{++}$	n.s.
Problem Solving	confidence	$\uparrow_{-}^{+++}$	<1%	$\uparrow_{+}^{++}$	<5%	$\downarrow_{++}^{++}$	n.s.	$\uparrow_{+}^{++}$	<5%
	pleasure	$\uparrow_{+}^{+++}$	n.s.	$\boxed{\downarrow_{+++}^{+++}}$	n.s.	$\downarrow_{+++}^{+++}$	n.s.	$\downarrow_{+++}^{+++}$	n.s.
	get through	$\downarrow_{-}^{+++}$	<1%	$\downarrow_{---}^{-}$	<1%	$\uparrow_{---}^{--}$	<1%	$\downarrow_{-}^{-}$	n.s.
	anxiety	$\downarrow_{---}^{++}$	<1%	$\downarrow_{--}^{-}$	<5%	$\boxed{\downarrow_{--}^{--}}$	n.s.	$\downarrow_{--}^{-}$	n.s.
	fear unexpected	$\downarrow_{--}^{++}$	<1%	$\downarrow_{--}^{++}$	<1%	$\uparrow_{--}^{-}$	n.s.	$\downarrow_{-}^{++}$	<1%
	correct answers	$\downarrow_{-}^{++}$	<1%	$\downarrow_{--}^{+}$	<1%	$\uparrow_{--}^{-}$	<1%	$\downarrow_{-}^{+}$	n.s.
	try new approach	$\uparrow_{+}^{+++}$	<1%	$\uparrow_{+++}^{+++}$	n.s.	$\downarrow_{+++}^{+++}$	n.s.	$\downarrow_{+++}^{+++}$	n.s.
	give up	$\downarrow_{--}^{++}$	<5%	$\downarrow_{---}^{-}$	<1%	$\downarrow_{---}^{--}$	n.s.	$\downarrow_{--}^{-}$	<5%

Table 5 : Desired changes compared with changes after problem-solving and after mathematics lectures

Note that *the attitudinal changes during the problem-solving course are all in the same direction as the desired change, with the exception of one*: "pleasure" was rated highly each time with positive attitudes changing only from 43 down to 42 (out of 44).

On the contrary, *all but one of the changes during the mathematics lectures are in the opposite direction*. Even the exception—"anxiety"—has an increase in those feeling anxious from 6 to 9, but the weighted average is biased marginally in the oppositive direction by the drop in "definitely not anxious" from 9 to 5.

During the problem-solving course, only four changes are not statistically significant: ***pleasure***, ***williness to work hard***, ***willingness to try a new approach*** remain highly rated, whilst ***mathematics is abstract*** has a small improvement from positive to negative.

Three items change significantly: ***ability to relate ideas*** and ***confidence*** both increase, whilst ***anxiety diminishes***. All other items have highly significant changes in the desired direction. Some beliefs are reversed so that after problem-solving students now believe that ***mathematics is more than facts and procedures,*** it involves ***inventing new ideas***, it ***makes sense***, it is ***not learnt just through memory***, there is ***less fear of the unexpected***, it is ***not just getting correct answers***. Others are greatly increased: ***mathematics is more about solving problems***, it can be ***understood more quickly***, and students are ***less likely to give up*** when encountering a difficulty.

However, six months later, after returning to the mathematics course many opinions have reverted back in the old direction. Of these there is a significant ***reduction in belief that mathematics is not just memorisation***, and highly significant ***reversal in belief that mathematics is just facts and procedures***; it is ***less about solving problems***, ***less about inventing new ideas***, ***less about doing the work for reasons other than to get through the course*** and ***less about things other than correct answers***.

Comparing the situation from before the problem-solving course with the status after six months back at regular mathematics lectures, many of the indicators revert back towards their old position. But three problem-solving attributes remain: ***confidence*** and ***unwillingness to give up*** remain significantly improved and ***fear of the unexpected*** is highly significantly reversed. Smaller changes are evident in the belief that mathematics ***make sense*** and that ***it is not necessary just to learn by memory***. (These are improved by a factor that would be significant at the 10% level, marked "n.s.*" in table 5.)

In addition to these changes, there are other items that are given at least "++" or "--" in the final ratings: ***mathematics is facts and procedures***, ***is about solving problems***, students ***work hard***, are ***able to relate ideas***, take great ***pleasure*** in their work, have ***low anxiety***, are ***willing to try a new approach***. All these are attributes carry over from earlier mathematics learning. The emphasis is on procedural aspects, working hard to solve problems and relate ideas to obtain pleasure and low anxiety. However, the comments of the lecturers earlier suggest that this pleasure is more the security of operating in a system set up to teach the students procedures which can be successfully tested than in developing flexible new skills appropriate for the changing modern world.

Student comments

The following selected comments written by the students in the final questionnaire bring to light several factors that could explain their changes in attitudes. In the perception of mathematics for instance, about a third (32%) reported that the regular mathematics did not allow them to think in a problem-solving manner:

> Since following the course I know mathematics is about solving problems. But whatever mathematics I am doing now doesn't allow me to do all those things. They are just more things to be remembered. male, year 5

> I believed mathematics is useful in that it helps me to think. Having said that it is hard to say how I can do this with the maths I am doing. Most of the questions given can be solved by applying directly the procedures we had just learned. There is nothing to think about. female, year 3

They saw that their mathematical training is rather rigid. They felt that their lecturers laid too much emphasis on content, and on unchallenging work:

> At the moment I am finding difficulty with maths because I am just not enjoying it. Too much emphasis is put on getting the right answer and not on method and understanding. female, year 4

> The mathematical atmosphere here is very bad; there is little discussion and it provides no encouragement to do maths. The content is emphasised over everything else. We are crammed full of lots of bland mathematical abstract theory. male, year 3

Some emphasise the way in which the lecturers move fast to complete the content:

> I did not enjoy most of the maths courses—too dependent on the lecturers. I don't find the way most of them teach particularly inspiring. We find ourselves hurrying through to keep up. There is no time to think about the mathematics we are doing. male, year 3

Some appreciate their knowledge in problem solving, suggesting it helps them to learn their mathematics and solve problems more effectively:

> The problem solving techniques help me come to terms with the abstract nature of the maths I am doing. I try to connect the ideas together and talk about them with my friends. It is not that easy though. But I felt all the effort worth it when I am able to do so. male, year 3

> I find the problem solving knowledge very useful in helping me understand the whys and the hows of advanced mathematics. It is much more satisfying than rote-learning. Furthermore it is actually easier to remember something that you understand. female, year 4

There are some who have minor reservations on their problem solving experience. But they believe it is necessary to have a positive attitude:

> The main disadvantage is time. It would take several hours maybe days to understand each new concept. Under the current circumstances we are finding ourselves rapidly hurrying to keep up. Sometime we were too bogged down in the technical details and we end up purely taking down the notes without even concentrating. This really defeats the problem-solving techniques. … But I think with further support from good teaching as well as tailoring the courses to suit the needs of the students the situation can be improved. male, year 5

Summary

Although lecturers prefer students to have a range of positive attitudes to mathematics, they expect the reality to be different. They prefer students to see mathematics as solving problems, making sense, with students working hard, able to relate ideas without needing to learn through memory, having confidence, deriving pleasure, with low anxiety and fear, ready to try a new approach and unwilling to give up easily on difficult problems. On the other hand, they expect them to see mathematics as abstract, failing to understand it quickly, not making sense, working hard to learn facts and procedures through memory, unable to relate ideas, with less confidence, obtaining less pleasure, working only to get through the course, with anxiety, fear, seeking only correct answers, and ready to give up when things get difficult.

By assigning a "desired direction of change" in the direction from what lecturers expect to what they prefer, it transpires that when doing a problem-solving course almost all the changes are in the desired direction and when returning to mathematics lectures, almost all the changes are in the reverse direction.

The findings show that the lecturers have little confidence in the students' ability to think mathematically and teach them accordingly. The students acquiesce to this approach, and set their sights on the lower target of learning procedurally to be successful in routine tasks. In this there is a widespread sense of pleasure although, after the problem-solving course, opinions expressed suggest concern that that the quantity and difficulty of the mathematics gives them little room for creative thinking.

Teaching problem-solving skills is not part of the lecturers' previous experience, consequently the lack of experience and the perceived difficulty of changing a formal system with so much content to be learned are severe deterrents to change. However, given the fact that problem-solving causes "positive changes in attitude" which are largely reversed in the standard course with its more difficult mathematical content, it is appropriate to pose the question:

> Given such a situation, do professors wish to continue to get what they expect, or do they want to change to attempt to get what they prefer?

References

Mohd Yusof, Y. & Tall, D. O. (1994): Changing Attitudes to Mathematics through Problem Solving. *Proceedings of PME 18*, Lisbon, IV, 401–408.

Skemp, R. R., (1979): *Intelligence, Learning and Action*, London: Wiley.

Kilpatrick, J. & Stannic, G. M. (1989): Historical perspectives on problem solving in the mathematics curriculum. In R. Charles & E. Silver (Eds.), *The teaching and asessing of mathematical problem solving*, (pp.1–22), Reston VA: NCTM.

Mason, J., Burton, L. & Stacey, K., (1982): *Thinking Mathematically*, London: Addison Wesley.