Qualitative Protocol Analysis

David Tall
University of Warwick, UK
david.tall@warwick.ac.uk

This paper appeared in the report of the research project: *Problem Solving Protocols: A Task-oriented Method of Analysis* by Joel Hillel and David Wheeler at Concordia University, Montreal, August 1982 as pp.178-181.

It is applied in pp. 182-186 to the problem square cutting (1, version ii), given on pp. 13, 14.

The paper here is laid out in 3 successive parts:

pp. 13, 14: The square cutting problem.
pp. 178-181 The draft proposal for the method of Qualitative Protocol Analysis
pp. 182-186 applying the analysis to a specific Protocol for Problem 1.ii.

Problem 1: "Square Cutting"

(open-ended version)	(i):	How many different ways can you cut up a square so that each piece you cut is a square?
(more explicit version)	(ii):	Show how to cut a square like this one into 9 square pieces. Could you cut it into 10 square pieces? 11 square pieces? Do you think it could be cut into any number of pieces?
("closed" version)	1(a):	Can you cut a square into four square pieces? Can you cut a square into seven square pieces? Can you cut a square into thirteen square pieces?

For all the above versions, a given square was drawn next to the problem statement.

This problem was used more frequently than the rest of the experimental problems. It is a problem requiring minimal mathematical knowledge (namely the notion of a square), provides many possible ways of generating some of the solutions and has an easy starting point, namely, making cuts in the form of a grid. However, this obvious starting point often leads to a "set" of thinking only in terms of squares of equal size and the challenge to the solver is to break out of such a "set". This problem was used frequently in the research because it is relatively easy to analyze the protocols since the solver's actions are overt (expressed as trial drawings).

From a theoretical point of view, the problem stands in contrast to the kind of "well-structured" problems used by information processing psychologists - it is not easy to identify the "given" or the "admissible operations" and consequently the problem has no single "state-space". In fact, discovering the "admissible operations" is part of the solver's task. Nor is the strategy of "means-end" analysis very useful here, since the solver cannot tell a priori whether she or he is approaching the goal or getting further away from it (except for the obvious cases of cutting into 4, 9, 16, etc.).

We initially used the "open-ended" version of the problem because we wanted to investigate such problems. However, we found that because of the numerous possibilities of generating "cuts", sessions tended to be lengthy, and it was difficult to compare across subjects. (The difficulty with open-ended problems is that there are potentially many different tasks which are embedded in them.)

The more explicit version ii), in which specific numbers of square pieces are asked for, followed by the more general question, was eventually dropped. The sessions tended to be rather lengthy, partly because divisions into 10 and 11 square pieces require different approaches (iteration and clustering for 10 and "bordering" for 11. See page 71 for elaboration). Towards the latter part of the research, only the "closed" version 1(a) was used. The subjects were asked to cut into 4 and 7 square pieces. If time allowed, the question about 13 square pieces was asked to see whether the solver recognized the generality of the method of iteration.

Totals: Version i) and ii): 12 protocols (including 4 adults)

Version 1(a) : 6 protocols.

DOCUMENT V

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David Tall
Mathematics Education Research Centre, Warwick University, U.K.

A protocol is the written report of a clinical interviewer. The analysis of such interviews presents a major problem, for in addition to factors pertinent to the particular interview, there must be general principles involved which apply to a broad class of interviews. Without such general principles an analysis would be merely anecdotal and non-generalizable.

How do we select these general principles? We could choose them according to various criteria: according to the mathematical content of the problem, or the nature of the thinking processes or other viewpoints. Clearly the nature of the mathematics is important, but an analysis based on this alone tends to superimpose a logical structure which is at variance with the observed manner in which thinking proceeds. For the moment we will concentrate on the qualitative nature of thinking which shows itself in a wide variety of protocols. The analysis is put forward as a topic for discussion. If it proves fruitful, refinements would be welcome, both in terms of the nature of the analysis itself and the names used for the various phenomena. (These are still in need of reorganisation and clarification.)

First let us begin with various observed phenomena:

1. Initial resonances: immediate responses to stimuli which occur without time for reflection. If consonant, such a resonance may provoke an ongoing schema but if dissonant elements are present to a sufficient degree there may be a conflict leading to an alternative schema of thought or a mental blockage.

2. Ongoing schemas: Connected chains of thought without major crises or changes in direction. In clinical interviews the extent of such schemas are sometimes difficult to determine because of the interjections of the interviewer. Sometimes the interviewer is asked to confirm a thought of the subject, in which case the interjection probably does not disturb the schema. Sometimes the subject is in such a strong chain of thought that the interviewer's interjection is not registered by the subject; once more the schema is unaffected. On the other hand a mere 'um huh' from the interviewer may have a disproportionate effect in changing the schema.

We may wish to distinguish certain types of schema, e.g. resonance

(RS) schema: carried on by the intense power of the thought process itself. Such a schema is not disturbed significantly by external comments. It may end up with the subject losing track of the original problem because the goal of solving the problem has been temporarily lost in the compulsion of the schema itself.

(SS) superposed schema: where the initial resonance involves two disparate schemas which produce a novel train of thought superimposing the two (or more) resonances. The subject may feel that such a superposition will lead to a solution and does not as yet sense a definite conflict.

(CS) conflict schema: the subject realises conflict is present but has not, as yet, resolved it. He may flash from one idea to another.

(E) other types: ? e.g. explanatory schema which is essentially post-solution (of at least part of the problem) and is running through the steps already thought out.

(OS) or: open search schema: prompted by an interviewer's question, not like resonance schema. Schemas are, broadly speaking, continuities in thought, but of course they may involve small discontinuities which are not noticed by an observer.

3. Qualitatively different are certain observable discontinuities:

(C) conflict: causing disruption of schema.

(B) mental blocks: stoppages in thought, often preceded by conflict.

(I) insights: sudden leaps in thought (which may be of the same nature as "initial resonances").

(F) finish of schematic action.

These discontinuities (particularly insights) are of extreme interest in clinical interviews. We need as much information as possible to interpret them (tape, audiovisual aids, etc.).

The flow is often affected by an interjection on the part of the interviewer. I've classified these into two subsets

(i) in response to a question (Q): (r) response to a yes or no question
(a) agree
(d) disagree
(n) neutral

(ii) interjections (c) seeking clarification
(q) questioning
(q') leading question
(p) prompting
(e) explaining
(g) grunt

Some random thoughts for discussion

1. Initial resonances often involve over extrapolation leading to later conflict - look for this. (marked X in protocol)

2. Pictorial and verbal evidence in protocols have different statuses. Pictures continue to be available in concrete form but verbal comments depend on memory of the subject. In particular, verbal comments in one schema may not be remembered in a later one. Thus conflicts in a written protocol may not be conflicts to the subjects at the time. CONFLICTS ARE LOCAL, NOT GLOBAL, PHENOMENA.

3. Does the memory of something earlier depend on the level of energy involved in the earlier mental activity in some way? (Mental activity involves electrical brain activity. Potential differences in different parts of the brain which lead to the passage of electrical current yield a potential function. Structurally stable brain activity may involve local minima of the potential function. The changes of equilibria in time may yield continuous schema of discontinuous leaps. Memory may depend in some way on the level of energy in the earlier activity.)

4. Local errors and corrections which don't interrupt the overall flow may give insight into the nature of brain activity. In ongoing schemas they may have little effect (a case of "structural stability?) but what about the effect of local errors near critical points of brain activity.

5. Conflict in problem solving protocols may be different from conflict in Piagetian tests. In problems the conflicts may be caused on occasion by transient electrical phenomena which when questioned cannot be recalled. In Piagetian transition there are usually two fairly stable (chemical) mental configurations causing the conflict; the subject is able to talk about each one separately in a suitable context.

6. Words like "carelessness", "mistake" etc. are a vocabulary from a logically based paradigm of interpretation. A'mental resonance' concept may be more suitable for description of thinking processes.

Analysis of Protocol 1.2

IR

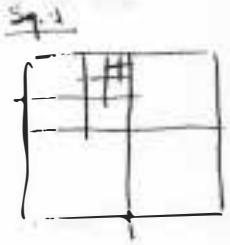
E: (Reads the problem). An infinite amount

(q) J: Infinite?

E: Amount of squares

(c/q) J: How would you do that?

E: You divide it by ... divide it into fours and then you divide each one of those into fours and you divide each one of those into fours again



At this stage J refines the problem because the initial resonance has lead to an impasse.

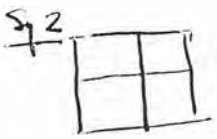
(e-q) J: Okay, I guess the question is: what are the numbers you would get, what possible numbers you would get in subdivisions?

(IR) E: You'll get ... whatchamacallit - multiples of 4

J: Multiples of 4?

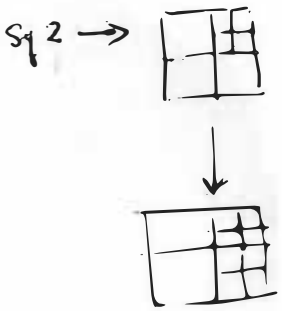
J can't help himself and asks for immediate clarification:

(q*) ... Show me how you would get 8



E corrects herself, makes another initial response which J questions, then goes into a powerful resonance schema which ends up out on a limb:

(C→IR) E: No ... wait a minute ... (not right) ... sorry, you would get powers of 4



(C) J: powers of 4?

(IR→C) E: Yea, you would get ... no actually you wouldn't (*draws here*) hold it (*thinking of drawing*) would you mind repeating the question?

(RS→B)

(e) J: Well, I am asking what are all the possible numbers

(IR) *(here E interjects: "Oh, o.k.", the rest of J's comment may not have been heard)*

that you can get through subdivisions like these

RS

E follows a strong (logical) resonance schema:

E: Well, 1 first of all, then 4, then you divide that into 4... then subtract 1 add 4, so we add 3 and do that each time. Okay, so you start with 4 and then you keep adding 3 every time you cut it again ... that makes up ... you can start with 1 and keep adding 3

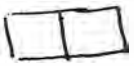
write
1+3+3+3+3

(F) *From E's tone of voice there is an air of finality about this response.*

(Q) J: Those are the only numbers that you can get?

This interjection prompts E to think intensely again, and go off at a tangent on a new track.

(OS?) (Q) E: — — — Wait a minute, you are allowed to have like ... some parts of squares and the rest rectangles or some kind of polygons? draws:



(e) J: No, every piece has to be a square

E responds immediately, returning to her original schema:

(F) E: Then those are the only one I can get

(as if to say: that's the finish of the problem)

(Q) J: Those are the only ones?

(F) E: Yea *(she is still sure.)*

(Q) J: You want to leave that as a final statement or do you want to ...

J's persistence finally unhinges her and she appears to be actively searching now. Something resonates - we are not sure what. Is this an "open search" schema or a "resonance schema" or what?

(OS?) E: Hum — — — — — — Do squares that appear as ... I guess they do count ... Oh dear (laughs), ... you can get 1 — — — well, it depends on how you look at it.

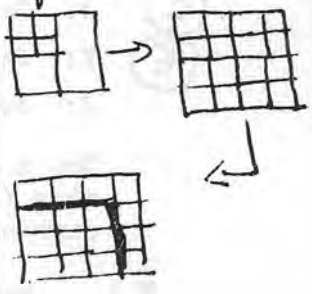
J prompts with a question in which it isn't clear what "it" is:

(C) J: Can you tell me what are the different ways to look at it?

E interprets the question in some meaningful way and launches into an explanatory schema, but towards the end of the explanation, her own words seem to prompt her to consider what she is talking about:

(E) E: Well, you can do it, say, taking one big square and a bunch of little squares

Sq. 4



(Q) J: Um huh *(has no effect)*

E: Or two big squares and a bunch of little squares ... or three.

(Q) J: What numbers would you get? *E continues thinking and may not hear question*

E: 1-2-3-4 ... still coming out ... well, it is ... I am probably missing something, but as far as I can see you still get the same

(F)

(q) J: Always 1, 4, 7?

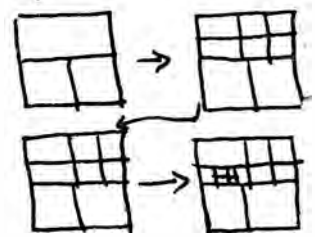
(F) E: 1, 4, 7, 11, I mean 10

(note immediate self correction which doesn't affect the broad flow)

(q*) J: Can you divide it into 9 square pieces?

E: (OS) — — — — — (laughs) (I) you can get anything ... okay, to amend the statement, I would say you can get any number of squares...

Sq.5

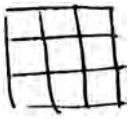


Note in the above J's question prompts an intense open search schema which leads to an initial resonance which is, in fact, an over extrapolation ("you can get anything"). Next J essentially repeats his previous question, and it becomes apparent from the time taken for E's reply that her previous schema had not solved the specific problem he posed (at least not stably enough to remember). She has to work out the specific solution anew:

(q*) J: How would you get 9?

(OS) E: Hum — — — — 3 lines, I mean 2 lines cutting across like a tic-tac-toe board

Sq.6



(q) J: Is that the same construction as you did before?

(E) E: ... No, I was cutting it in halves before

J then checks on an earlier picture:

(q) J: Um huh ... let me ask you this: that square you have at the lower right hand corner there — *(Sq.5)*

(E) E: I don't know that those are real squares

J: You don't know that they are squares .. <u>you just</u> ...

(q) *Now J tries to guide in another direction, with dramatic results*

is there another number you think you can divide it into?

immediate resonance (possibly from the picture  *):*

(IR) E: ... Aaa ... 3 times 3 is 9, 4 times 4 would be 16, 5 times 5 would be 25

(C) J: Um huh, what are these numbers?

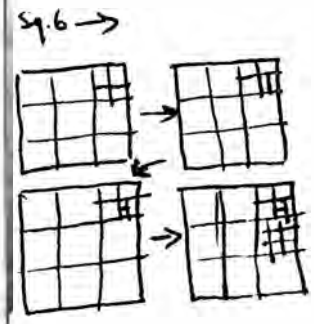
(E) E: Square numbers ... squares of integers

J: You can always cut them into square numbers ...

Now a contradiction provoking "torpedo", although (as J notes) this is actually part of 1 + 3 + 3 ... schema, but <u>not</u> of current schema with which it is in <u>local</u> conflict.

(q*) hum, I claim you can divide it into 13 square pieces

provokes open search schema, which leads to a questioning of the way the counting of squares is performed, perhaps there is a different way of counting to get 13.



E. 13 — — — — — — — — — — does it count if you have one big square and 4 little squares in it?

J: No

E: Does it count as 5 or just 4?

J: No, just 4. You could count it as 5 if you were playing the game that way, but we are just counting the pieces ...

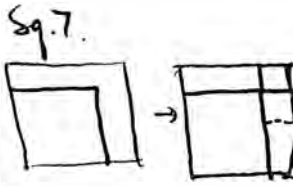
J then asks for an explanation which E gives and returns to her train of thought:

what were you trying in the previous picture?

E: Well taking a big one and a lot of little ones, but without having side measurements I couldn't tell whether I was doing squares or rectangles

J: Um huh

E: ... 13 ... 13 minus 9 is 4 — — — —

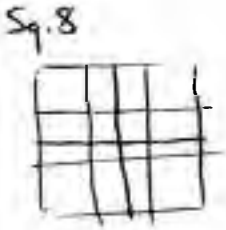


Now E has a new idea

What about lines cutting through other squares?

J: How do you mean?

Perhaps now the "13 minus 9" has led to a superposition of resonances:



E: Lets see, a square with ... you make the ... the tic-tac-toe board and then you cut it in ½ just as in normal ... you cut it into 9 parts and you cut it into 4 parts and the lines across...Oh that's not...hum (laughs)...can you do that?

She hits a conflict here causing question

J: Well, would all the pieces be squares in that picture?

J's response causes E to reconsider:

E: That wouldn't be, but that would ... alright

At this point she appears already to have decided to abandon the idea, (but it reappears shortly which shows she has not completely resolved the conflict). Meanwhile I don't understand J's next comment E also gives no audible reaction - she is off on another open search schema.

J: As long as you are counting just the square pieces, it is okay

E: — — can add 3 — — — — so I could do 10 ... I can't do 10 and

From E's tone of voice, it is clear she is following a logical chain of thought of the "if I could do x, then I could do y" variety. J asks for clarification:

C J: How is that again?

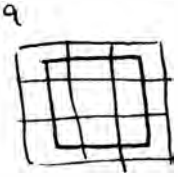
E E: I could add 3 squares to any diagram so I am working backwards from the problem

C J: I see

Now here J is familiar with the problem: he sees that E is close to a solution in the schema 13 = 1 + 3 + 3 + 3 + 3. But E is in a different ongoing schema and does not recall the earlier one at this point. She continues to follow her current line of thought, subsides into open search (?) schema and resonates with the previous superposition schema which clearly was not satisfactorily terminated.

RS → SS E: If I can add 3 to anything and if I can get 10 then I can get 13 — — — — — — — — — — one minute, I'll do something

Sq 9

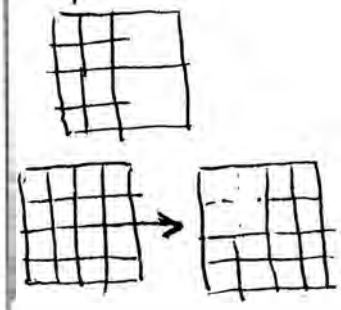


C J: You'll do what?

E: Well, if you can make a square into 4 and you can make a square into 9, I am trying to add the two together and get something

She continues to think, but clearly the superposition schema has seeds of conflict and the "add 3" schema is more resonant. E returns to it.

Sq 10



RS E: __ __ __ __ __ __ *(45 secs.)* (to herself) if I can add 3 I can subtract 3. 1, 2, 3, ..., 13

C J: If you can add 3?

E E: If I can add 3 to anything I can subtract 3 from anything and 16 is also a perfect square so I drew 16 little squares and made 4 squares into 1

J: Good, that is a very nice way to solve it.

The conversation actually continues. It becomes clear that the "add three", "subtract theree" schemas haven't yet been thoroughly worked out, i.e. the resonance that suggested " + 3" implies " - 3" is an over-extrapolation. The rest of the interview refines this concept and looks at all possible numbers of squares, without fully solving the problem.