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GROUP REPRESENTATIONS, λ-RINGS AND THE J-HOMOMORPHISM

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INTRODUCTION

THIS paper arose from a desire to apply the work of J. F. Adams 'on the groups $J(X)$' [2] to the case where X is the classifying space B_G of a finite group G. Since Adams' calculations apply only to a finite complex X, and B_G is infinite, the results could not be applied directly. Rather than quoting theorems and using limiting processes, the pure algebra has been isolated and independently reworked in such a way that it not only applies to the situation considered, but is also of general interest. This occurs in Parts I and III. The algebra used requires knowledge of special λ-rings (which arise in K-theory and elsewhere). Part I is a self-contained study of these. (A special λ-ring is a commutative ring together with operations $\{\lambda^n\}$ having the formal properties of exterior powers). Part III contains the main algebraic theorem, which readers familiar with the work of Adams [2] may recognise as essentially including a proof of his theorem '$J'(X) = J''(X)$'.

Arising from the applications of this theory, the principal theorem of the paper lies in another field of study, in the topology of group representations. Broadly speaking, a general type of problem that may be posed is this: given two representations E, F of a group G, what constraints are imposed on E and F by the existence of a given type of map between them which commutes with G-action? More precisely, if E, F are unitary (or orthogonal) representations, so that the unit spheres $S(E)$, $S(F)$ are preserved by G-action, under what conditions can there exist a G-map $\phi : S(E) \to S(F)$ where ϕ is a diffeomorphism, homeomorphism, homotopy equivalence or some other given type of map?

Some results are known, for example de Rham [17] has shown that if ϕ is a diffeomorphism, then E, F must be isomorphic representations.

In this paper, only very weak restrictions are placed on ϕ. The following theorem is proved: