HOMEWORK 6, MATH 114, SPRING 2003

DUE TUESDAY MAY 27

(1) Give an explicit matrix S such that $A = SA'S^{-1}$ (where A' is the transpose of A) for the matrix
$$A = \begin{pmatrix} 3 & 1 & 0 & 0 \\ 0 & 3 & 1 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 4 \end{pmatrix}.$$
(2) Give an algorithm to find a matrix S such that $A = SA'S^{-1}$ for any matrix A.
(3) Find the rational canonical form J of the matrix
$$B = \begin{pmatrix} 3 & 1 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 4 \end{pmatrix},$$
and a matrix S such that $J = SBS^{-1}$.
(4) Show that if A is a matrix for which the minimal polynomial $m_A(x)$ has degree r, then there is a vector $v \in k^n$ such that $\{v, Av, \ldots, A^{r-1}v\}$ are linearly independent, but no vector w for which $\{w, Aw, \ldots, A^r w\}$ are linearly independent.
(5) Does the rational canonical form of A depend on the field k? In other words, if $k \subseteq k'$, and A has entries in k, is the the rational canonical form of A viewed as a matrix with entries in k the same as the rational canonical form of A viewed as a matrix in k'? You may find it easier to think about if $k = \mathbb{Q}$ and $k' = \mathbb{C}$, but your answer should work for any pair of fields $k \subseteq k'$.