

HOMEWORK 5, MATH 114, SPRING 2003

DUE TUESDAY MAY 20

- (1) Find the Jordan canonical forms of the following matrices. What are their minimal polynomials? Also find the change of basis matrix S which transforms each matrix into its Jordan canonical form. You may use a computer to help compute generalized eigenvectors. Do *not* use a “Find the Jordan canonical form” command except to check your answer.

(a)

$$A = \begin{pmatrix} 3 & -1 & -1 \\ 1 & 1 & -1 \\ 2 & -1 & 0 \end{pmatrix}.$$

(b)

$$A = \begin{pmatrix} -2 & 1 & 0 & 0 & 0 & 0 \\ -9 & 4 & 0 & 0 & 0 & 0 \\ -1 & 7 & -2 & 1 & 0 & 0 \\ 57 & 1 & -9 & 4 & 0 & 0 \\ -76 & -8 & 15 & -5 & 1 & 0 \\ 188 & 30 & -42 & 14 & 1 & 2 \end{pmatrix}.$$

- (2) What is the rational canonical form of the first matrix above?
- (3) Show that if $\ker(A - \lambda I)^k = \ker(A - \lambda I)^{k+1}$ then $\ker(A - \lambda I)^r = \ker(A - \lambda I)^k$ for all $r \geq k$. Use this to describe an algorithm to compute $N(\lambda)$.