HOMEWORK 4, MATH 114, SPRING 2003

DUE TUESDAY MAY 6

(1) Recall the *Smith Normal Form* of a $d \times n$ integer matrix A is a $d \times n$ integer matrix D such that $d_{ij} = 0$ for $i \neq j$ and $d_{(i+1)(i+1)}$ divides d_{ii} for $1 \leq i \leq \min(d, n)$, and integer matrices U and V with determinant one such that $D = UAV$. Find the Smith Normal form of
$$\begin{pmatrix} 4 & 3 & 2 \\ 6 & 7 & 9 \end{pmatrix}.$$
Hint: You can use a computer algebra system such as Maple to check your answer. If you use Maple before you've otherwise solved the problem, factor the matrices U and V into the product of elementary matrices (those coming from row or column operations).

(2) Show that if A is a 2×2 matrix, it has a Smith Normal Form (ie show that such a matrix D and matrices U and V exists). You do not need to show that it is unique. (Though if you've taken a group theory class, you should see if you can prove it!)

(3) Use Question 2 to show that any $d \times d$ matrix has a Smith Normal Form.

(4) Compute all the circuits of
$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 & 4 \end{pmatrix}.$$

(5) Lax, Chapter 6, Exercise 1.