MA5Q6 GRADUATE ALGEBRA - HOMEWORK 2

DUE TUESDAY 23/10, 12PM

Hand in all questions to Carole Fisher's office. You are encouraged to work together on the problems, but your written work should be your own. Write on your homework the names of the people you worked with, and any references you consult.

(1) Show that for all objects C in a category $\mathcal{C}$ the element $1_C \in \hom(C, C)$ is unique.
(2) Show that if A, B are isomorphic objects of a category $\mathcal{C}$ and $T : \mathcal{C} \to \mathcal{D}$ is a functor, then $T(A)$ is isomorphic to $T(B)$.
(3) Let $(\mathcal{P}, \leq)$ be a partially ordered set (ie $\leq$ is reflexive, anti-symmetric, and transitive). Show that $\mathcal{P}$ can be regarded as a category, where the objects are the elements of $\mathcal{P}$, and $\hom(A, B)$ is a singleton set if $A \leq B$, and empty otherwise.
(4) Let T be the function that takes a group to its abelianization (the quotient by the commutator subgroup generated by all products $g^{-1}h^{-1}gh$). Show that this "is" a functor from Groups to Abelian Groups (ie show that there is a corresponding function on morphisms that together with this function on objects makes a functor).
(5) Is the function T that takes a group G to its centre a functor? (ie is there a corresponding function on objects that makes this into a functor?) (Recall that the centre is the set $\{g \in G : gh = hg \text{ for all } h \in G\}$).
(6) Is the function T that takes a finite set S to a finite-dimensional vector space over $\mathbb{C}$ with basis indexed by S a functor?
(7) Give two examples of categories and two functors that were not covered in lecture (or above). In each case prove that they are categories and functors. PhD students may want to ask their supervisors for suggestions.