

## MATH 559 HOMEWORK 4

DUE: MONDAY MARCH 26

All rings  $R$  are commutative with 1, and if not otherwise noted  $M$  and  $N$  are  $R$ -modules. Warning: I don't have the most recent printing of Eisenbud - if the "name" of an exercise doesn't coincide with its number, please let me know immediately. For Eisenbud "graded" means  $\mathbb{Z}$ -graded unless otherwise stated.

- (1) Eisenbud 4.1 (Extension of Corollary 4.5 to modules)
- (2) Eisenbud 4.11 (Projective modules over a local ring are free)
- (3) Eisenbud 4.12 (Projective modules are flat)
- (4) Eisenbud 4.14 (Integral closures of ideals)
- (5) Eisenbud 4.15 (Examples of integral closures of ideals).
- (6) Find the integral closure of the ideal  $\langle x^2, y^2 \rangle \subset \mathbb{k}[x, y]$ . Let  $R = \mathbb{k}[x^3, y^3] \subset \mathbb{k}[x, y]$ , and  $S = \mathbb{k}[x^3, x^2y^2, y^3]$ . Is  $S$  an integral  $R$ -algebra?
- (7) Eisenbud 4.21 (Graded integral closure)
- (8) Eisenbud 4.30 and 4.31 (a proof of the Nullstellensatz for uncountable fields). You may substitute 4.32 for 4.31 if you want, or do 4.29 instead of either, with the proviso that your proofs of these must be very clear.
- (9) Let  $R$  be a  $\mathbb{Z}$ -graded ring with  $R_0$  a field. Many things that are true for local rings are also true for  $R$ .
  - (a) (Graded Nakayama). Let  $M$  be a graded  $R$ -module, and let  $I$  be a homogeneous ideal generated by elements of positive degree. Then if  $IM = M$  we have  $M = 0$ .
  - (b) If  $M$  and  $N$  are graded  $R$ -modules with  $M \otimes_R N = 0$ , then  $M = 0$  or  $N = 0$ .