MATH 551 HOMEWORK 8

DUE WEDNESDAY, NOVEMBER 9

You are encouraged to work on the homework together, but your final write-up should be your own. Please write down on your homework the name of any collaborators. No late homework will be accepted. "Hungerford I.1.3" means Question 3 in the exercises at the end of Section 1 of Chapter 1.

(1) Let R be a commutative ring with identity 1_R.
   (a) Show that the ideal $\langle a_1, \dots, a_n \rangle$ for $a_i \in R$ is $\{r_1 a_1 + \cdots + r_n a_n : r_i \in R, 1 \leq i \leq n\}$. (Do not cite results from the book we have not discussed in class).
   (b) Let A be an R-module, and let B be the (left) submodule generated by $a \in A$. Show that $B = \{ra : r \in R\}$.
   (c) Give a counterexample to the first part if R is not commutative. Did you need commutativity in the second part?

(2) Give an example of a ring R and two sets $A, B \subseteq R$ for which $AB \neq \{ab : a \in A, b \in B\}$.

(3) Let R be a ring with identity. An R-module A is *cyclic* if it is generated by one element. Show that a cyclic R-module is isomorphic to R/J, where J is a left ideal of R.

(4) Hungerford IV.1.5

(5) Hungerford IV.1.9. What linear algebra concept does this generalize?

(6) **Fall 2002** Let p be an odd prime, and let R be the ring of "p-quarternions"; that is

$$R = \{a_0 + a_1 i + a_2 j + a_3 k : a_n \in \mathbb{Z}/p\mathbb{Z} \text{ for } n = 0, 1, 2, 3, \text{ and}$$
$$i^2 = j^2 = k^2 = -1, ij = k = -ji, jk = i = -kj, ki = j = -ik\}.$$

   Show that R is a simple ring, so the only two-sided ideals of R are the zero ring and R itself.