

## MATH 551 HOMEWORK 4

DUE WEDNESDAY, OCTOBER 5

You are encouraged to work on the homework together, but your final write-up should be your own. Please write down on your homework the name of any collaborators. No late homework will be accepted. “Hungerford I.1.3” means Question 3 in the exercises at the end of Section 1 of Chapter 1.

- (1) Hungerford I.9.3
- (2) Hungerford I.9.8
- (3) Hungerford II.1.2
- (4) Hungerford II.1.10
- (5) Hungerford II.1.11
- (6) Show that the group of automorphisms of  $\mathbb{Z}^n$  is isomorphic to the group of  $n \times n$  matrices with integer entries and determinant  $\pm 1$ .
- (7) **Written Qualifying Exam, Fall 2004:** Let  $G$  be the set of all complex  $3 \times 3$  matrices which have exactly one nonzero element in every row and in every column. Show that  $G$  is a group under matrix multiplication. Show that  $G$  has two normal subgroups  $G_1$  and  $G_2$  with  $G_1 \subset G_2 \subset G$  such that  $G_1$ ,  $G_2/G_1$  and  $G/G_2$  are all abelian groups. (The original question let you assume that  $G$  is a group).