# MATH 551 HOMEWORK 2

DUE WEDNESDAY, SEPTEMBER 21

You are encouraged to work on the homework together, but your final write-up should be your own. No late homework will be accepted. "Hungerford I.1.3" means Question 3 in the exercises at the end of Section 1 of Chapter 1.

(1) **The symmetric group**
   (a) Convince yourself that every element of $S_n$ can be written as a product of disjoint cycles, and this representation is unique up to the order of the cycles. See Hungerford, Section I.6 if necessary. You do not need to hand this in.
   (b) Show that the adjacent transpositions $\{(ii+1) : 1 \leq i \leq n-1\}$ generate $S_n$ (that is, every element of $S_n$ can be written as a product of finitely many adjacent transpositions).
   (c) If $\sigma \in S_n$ can be written $\sigma = \tau_1\tau_2\tau_3$, where the $\tau_i$ are adjacent transpositions, we say this expression for $\sigma$ has length three. The length of $\sigma \in S_n$ is the length of the shortest expression for $\sigma$. Which element(s) of $S_n$ has/have the longest length?
   (d) A trivial way to get different expressions for the same element of $S_n$ is to add $\tau\tau$ to an expression. For example, $(12) = (13)(13)(12)$. If we require that there are no adjacent repeated adjacent transpositions in an expression for $\sigma$, is the expression unique?
   (e) Show that the transposition $(12)$ and the $n$-cycle $(123\ldots n-1n)$ generate $S_n$.

(2) Hungerford I.2.15.
(3) Hungerford I.2.19.
(4) Hungerford I.3.3.
(5) Hungerford I.1.7, Hungerford I.4.4
(6) Hungerford I.5.10.
(7) Hungerford I.5.15.
(8) Show that if $H$ and $K$ are subgroups of a group $G$, then every element of $H \vee K$ can be written in the form $h_1k_1h_2k_2\ldots h_rk_r$ for some $h_i \in H, k_i \in K$.