MATH 551 HOMEWORK 11

DUE WEDNESDAY, NOVEMBER 30

You are encouraged to work on the homework together, but your final write-up should be your own. Please write down on your homework the name of any collaborators. No late homework will be accepted. "Hungerford I.1.3" means Question 3 in the exercises at the end of Section 1 of Chapter 1.

(1) Hungerford, III.3.1
(2) Hungerford, III.3.3
(3) Hungerford, III.3.4
(4) **Spring 2002** Call an integral domain R *special* if and only if the intersection of any two principal ideals in R is again principal (generated by one element).
   (a) Show that if R is special, then any two nonzero elements of R have a greatest common divisor. (An element $a \in R$ is the greatest common divisor of $b, c \in R$ if a divides b, a divides c, and if $d \in R$ divides both b and c, then d divides a.)
   (b) Give an example of a special integral domain R and nonzero elements $a, b \in R$ such that the greatest common divisor of a and b is not an R-linear combination of a and b.
(5) **Spring 2001** Let A be the abelian group of quintuples of integers under addition, and let G be the subgroup generated by the elements

$$(1, -1, 0, 2, 1), (2, 3, 1, 1, 0), (4, 1, 0, 0, 2), (-1, 1, -1, 1, 1), (1, 1, 1, 1, 1).$$

   Determine the group A/G as a product of cyclic groups.
(6) Describe an algorithm to answer all questions of the form of the previous question. Hint: You want a computational form of our theorem about subgroups about free abelian groups.
(7) **Berkeley Prelim Spring 1999** Let G be a finite group with identity e. Suppose that for all $a, b \in G$ distinct from e there is an automorphism σ of G such that $\sigma(a) = b$. Prove that G is abelian. (Added): Give an example of such a group.