MATH 244, SECTIONS 05, 06, 08

SAMPLE MIDTERM

The following questions (mostly taken from the book) illustrate the scope and level of difficulty of the upcoming midterm. One caveat: This is longer than the actual midterm, though you should be able to do it in under two hours.

(1) Do questions 15–20 of Section 1.1 on matching direction fields and equations.

(2) Solve the initial value problem $ty' + 2y = t^2 - t + 1$, $y(1) = 1/2, t > 0$.

(3) Solve the initial value problem $y' = (1 - 2x)/y$, $y(1) = -2$. Where is the solution valid?

(4) A tank with a capacity of 500 gallons originally contains 200 gallons of water with 100 pounds of salt in solution. Water containing one pound of salt per gallon is entering at a rate of three gallons per minute, and the mixture is allowed to flow out of the tank at a rate of two gallons per minute. Find the amount of salt in the tank at any time prior to the instant when the solution begins to overflow. Find the concentration (in pounds per gallon) of salt in the tank when it is on the pount of overflowing. Compare this concentration with the theoretical limiting concentration if the tank had infinite capaacity.

(5) Consider the differential equation $y' = y^2(1 - y)^2$. Find all equilibrium solutions. For each such solution, decide if it is asymptotically stable, unstable, or semistable.

(6) Show that the equation $y + (2x - ye^y)y' = 0$ is not exact, but becomes exact after multiplying by the integrating factor $\mu(x, y) = y$. Use this to solve the equation.

(7) Consider the initial value problem $y' = 2y - 3t$, $y(0) = 0$. Write down the formula for the backward Euler method, and rearrange to get a formula for y_1 that only depends on y_0, t_1 and h. What is the one-step estimate for $y(0.1)$ using the formula? (Write your answer as an expression ready to enter into your calculator.)

(8) Solve the initial value problem $y'' + y' - 2y = 0$, $y(0) = 1, y'(0) = 1$. Sketch a graph of the solution and describe its behaviour as t increases.

(9) Compute the Wronskian $W(y_1, y_2)(\theta)$ of $y_1(\theta) = \cos^2(\theta)$ and $y_2 = 1 + \cos(2\theta)$.

(10) Find the largest open interval containing $t = 3$ on which a solution to the 1w initial value problem $t(t - 4)y'' + 3ty' + 4y = 2$, $y(3) = 0, y'(3) = -1$ is guaranteed to exist. Do not attempt to solve the equation.