MA 243 HOMEWORK 8

DUE: THURSDAY, 29, 2007, BY 12PM

Hand in the problems in Section B *only* to the boxes outside the undergraduate office. You are encouraged to work together on the problems, but your written work should be your own.

A : Warm-up problems

(1) Consider the line $L = \{y = 0\} \cap \mathbb{H}^2$ in $\mathbb{H}^2$. Show that this corresponds to the line $y = 0$ in the Poincaré disk model. (See QB2 for the extension).

(2) Let E be the affine subspace of $\mathbb{A}^2$ given by $E = \{(1, 0) + \lambda(2, 3)\}$. Find $\mathbf{u}_0, \mathbf{u}_1$ so that $E = \{\mu_0\mathbf{u}_0 + \mu_1\mathbf{u}_1 : \mu_0 + \mu_1 = 1\}$. Find a matrix C and vector $\mathbf{d}$ so that $E = \{\mathbf{x} \in \mathbb{R}^2 : C\mathbf{x} = \mathbf{d}\}$ (Hint: vectors of size one are numbers!)

B: Exercises

(1) In this question we will show that the circumference of a circle of radius r in S^2 is $2\pi \sin(r)$. As in $\mathbb{E}^2$, a circle of radius r with centre P is the set of all points in S^2 distance r from P.
   (a) If ΔPQR is a triangle with $d(P, Q) = d(P, R) = r$, and the angle between PQ and PR at P equal to α, what is the distance $d(Q, R)$?
   (b) Use this to compute the circumference of a regular polygon in S^2 with n vertices and radius r by dividing the polygon into triangles around a central vertex.
   (c) Write an expression for the circumference of the circle of radius r as a limit as n goes to infinity. Evaluate this limit.

(2) Consider the line $L = \{t = 2x\} \cap \mathbb{H}^2$ in $\mathbb{H}^2$. Show that in the Poincaré disk model of $\mathbb{H}^2$, L is taken to an arc of the circle of radius $\sqrt(3)$ centred at the point $(2, 0)$ (using the identification of projecting from $(-1, 0, 0)$ as in the diagram on p70 of Cannon, Floyd, Kenyon and Parry, which is linked on the main webpage under announcements).

(3) Let E be the affine subspace of $\mathbb{A}^3$ given by $E = \{(1, 0, 0) + \lambda_1(1, 2, 3) + \lambda_2(4, 5, 6) : \lambda_1, \lambda_2 \in \mathbb{R}\}$. Find $\mathbf{u}_0, \mathbf{u}_1, \mathbf{u}_2$ so that

$E = \{\mu_0\mathbf{u}_0 + \mu_1\mathbf{u}_1 + \mu_2\mathbf{u}_2 : \mu_0 + \mu_1 + \mu_2 = 1\}$. Find a matrix C and vector $\mathbf{d}$ so that $E = \{\mathbf{x} \in \mathbb{R}^3 : C\mathbf{x} = \mathbf{d}\}$.

C: Extensions

(1) Repeat the calculation of exercise B1 for $\mathbb{H}^2$ What answer do you get? What happens when you do this in $\mathbb{E}^2$?

(2) (Highly recommended!!!) There are many many interesting problems in the notes/book we did not have time to cover. For example, prove the spherical or hyperbolic sine law. Repeat the previous exercise for the area of a spherical/hyperbolic circle.