MA 243 HOMEWORK 4

DUE: THURSDAY, NOVEMBER 1, 2007, BY 12PM

Hand in the problems in Section B *only* to the boxes outside the undergraduate office. You are encouraged to work together on the problems, but your written work should be your own.

A : Warm-up problems

1. Decompose the motion
$$T(\mathbf{x}) = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \mathbf{x} + \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$
of $\mathbb{E}^2$ into a product of reflections.
2. Show that every rotation in $\mathbb{E}^2$ can be written as the composition of two reflections. (Hint: Last week's homework).

B: Exercises

1. Prove that if triangles ABC and $A'B'C'$ have $d(A, B) = d(A', B')$, and the angles at A and B equal the angles at A' and B' respectively: $\angle BAC = \angle B'A'C', \angle ABC = \angle A'B'C'$, then ABC is congruent to $A'B'C'$. Use the language and definitions of this module.
2. Let T be the motion of $\mathbb{E}^3$ given in coordinates by $T(x_1, x_2, x_3) = (\mathbf{x}) = (-x_3, -x_2, x_1)$. Write T as the composition of rotations and reflections and a translation as described in class. Then write T as the composition of at most four reflections as described in class.
3. Write an equation for the perpendicular bisector Π of the line between $(2, 0, 0)$ and $(2, 1, 3)$. Write down in coordinates the motion of reflecting in the plane Π.

C: Extensions

1. (a) Prove that if $T(\mathbf{x}) = A\mathbf{x} + \mathbf{g}$ is a motion, then $\det(A) = \pm 1$.
   (b) Show that the value of $\det(A)$ does not depend on the choice of coordinates (so the notions of orientation preserving/reversion are well-defined).

2. We've often used the fact that given a Euclidean frame $P_0, P_1, \ldots, P_n$ there is choose of coordinates that takes P_0 to $\mathbf{0}$ and P_i to $\mathbf{e}_i$ (the *standard Euclidean frame*). (This is why we can choose coordinates so a rotation is about the origin, for example, or a reflection is about the xy-plane). Prove this rigourously.