MA 243 HOMEWORK 1

DUE: THURSDAY, OCTOBER 11, 2007, BY 12PM

Hand in the problems in Section B *only* to the boxes outside the undergraduate office. You are encouraged to work together on the problems, but your written work should be your own. Note that this first assignment is somewhat unusual in the questions A1 and B1.

A : Warm-up problems

(1) Who was Lobachevsky?
(2) Show that the set $\{(x, y) \in \mathbb{R}^2 : 3x + 4y = 3\}$ is a line by giving vector $\mathbf{u}, \mathbf{v}$ from lecture.
(3) Let $\mathbf{u} = (1, 0)$ and $\mathbf{v} = (2, 3)$. Write an equation for the line $\{\mathbf{u} + \lambda\mathbf{v} : \lambda \in \mathbb{R}\}$.
(4) Describe the line joining the points $(1, 2)$ and $(2, 1)$ both parametrically, and by equations.

B: Exercises

(1) What is the parallel postulate? Cite any source you consult. Internet sources are ok for this question, though often not for scholarly work. Write a short paragraph (approximately five to ten sentences) summarizing what you discover about the history of this axiom.
(2) Describe the line joining $(1, 0, 2)$ and $(-1, 3, 1)$ in $\mathbb{R}^3$ both parametrically, and by equations. (Hint: You may need more than one equation).
(3) We showed in class that the triangle inequality holds. Now show the converse. Specifically, suppose that $a, b, c \in \mathbb{R}$ satisfy

$$0 < a < b + c, \quad 0 < b < a + c, \quad 0 < c < a + b.$$

Give a geometric construction (say, with ruler and compass) to prove that there exists a triangle in $\mathbb{R}^2$ with sides of length equal to a, b, c.

What special thing happens in your construction if one of the inequalities becomes equality, say, $a = b + c$?
(4) What can you conclude if $\mathbf{x}, \mathbf{y}, \mathbf{z} \in \mathbb{R}^n$ with $\mathbf{x} \in [\mathbf{y}, \mathbf{z}]$, $\mathbf{y} \in [\mathbf{x}, \mathbf{z}]$, and $\mathbf{z} \in [\mathbf{x}, \mathbf{y}]$? Justify your answer.

C: Extensions

(1) If we want to describe a line in $\mathbb{R}^n$ by equations, how many equations do we need? What do they look like? (for example, what degree are they?) What does this have to do with your linear algebra module?

(2) Our distance function $d(\mathbf{x}, \mathbf{y}) = |\mathbf{x} - \mathbf{y}|$ comes from the *norm* $|\mathbf{x}| = \sqrt{\mathbf{x} \cdot \mathbf{x}} = \sqrt{\sum_{i=1}^{n} x_i^2}$. Two other norms on $\mathbb{R}^n$ are given by $|\mathbf{x}|_1 = \sum_{i=1}^{n} |x_i|$, and $|\mathbf{x}|_\infty = \max_{i=1}^{n} |x_i|$. We can then define $d_1(\mathbf{x}, \mathbf{y}) = |\mathbf{x} - \mathbf{y}|_1$ and $d_\infty(\mathbf{x}, \mathbf{y}) = |\mathbf{x} - \mathbf{y}|_\infty$.

(a) Calculate $d_1(\mathbf{x}, \mathbf{y})$ and $d_\infty(\mathbf{x}, \mathbf{y})$ for $\mathbf{x} = (1, 0, 0)$ and $\mathbf{y} = (-1, 2, 3)$.

(b) The metric d_1 is often called the "Manhattan metric". Can you see why?