

## MA 243 HOMEWORK 8

DUE: THURSDAY, DECEMBER 4 2008, BY 12PM

Hand in the problems in Section B *only* to the boxes outside the undergraduate office. You are encouraged to work together on the problems, but your written work should be your own.

### A : WARM-UP PROBLEMS

- (1) Find a projective transformation of  $\mathbb{P}^1$  taking the set  $\{(1 : 0), (0 : 1), (1 : 1)\}$  to  $\{(1 : 2), (1 : 3), (1 : 4)\}$ . Repeat for the sets  $\{(1 : 0), (1 : 2), (1 : 3)\}$  and  $\{(0 : 1), (1 : 1), (1 : 5)\}$ .
- (2) Compute the cross-ratio  $\{P, Q; R, S\}$  for points  $P, Q, R, S$ :
  - (a)  $\{[1 : 0], [0 : 1], [1 : 1], [1 : 3]\}$ ;
  - (b)  $\{[0 : 1], [1 : 0], [1 : 3], [1 : 1]\}$ ;
  - (c)  $\{[1 : 1], [0 : 1], [1 : 0], [1 : 3]\}$ .

### B: EXERCISES

- (1) Find a projective transformation of  $\mathbb{P}^2$  taking the (ordered) list  $\{(1 : 1 : 0), (1 : 0 : 1), (1 : 1 : 1), (0 : 1 : 1)\}$  of points to the (ordered) list  $\{(1 : 0 : 0), (0 : 1 : 0), (0 : 0 : 1), (1 : 1 : 1)\}$ .
- (2) Compute the cross-ratio  $\{P, Q; R, S\}$  of the set  $\{P = (1 : 0), Q = (1 : 1), R = (2 : 1), S = (1 : 2)\}$  of points in  $\mathbb{P}^1$ .
- (3) Recall that we embed  $\mathbb{A}^n$  into  $\mathbb{P}^n$  by sending  $\mathbf{x}$  to  $(1 : \mathbf{x})$ . Given an affine transformation  $T(\mathbf{x}) = A\mathbf{x} + \mathbf{b}$ , write down the corresponding projective transformation it extends to (this was given in class briefly). Let  $S(\mathbf{x}) = A'\mathbf{x} + \mathbf{b}'$ . Write down the composition  $S \circ T$ , and compare it with the result of composing the corresponding projective transformations.
- (4) Read the Proposition in section 5.6 of the notes. Suppose that the cross ratio  $\{P, Q; R, S\} = \lambda$ . There are twenty-four permutations (bijections)  $\pi : \{P, Q, R, S\} \rightarrow \{P, Q, R, S\}$ . How many different values does  $\{\pi(P), \pi(Q); \pi(R), \pi(S)\}$  take? Hint: See exercises to Chapter five in the notes.